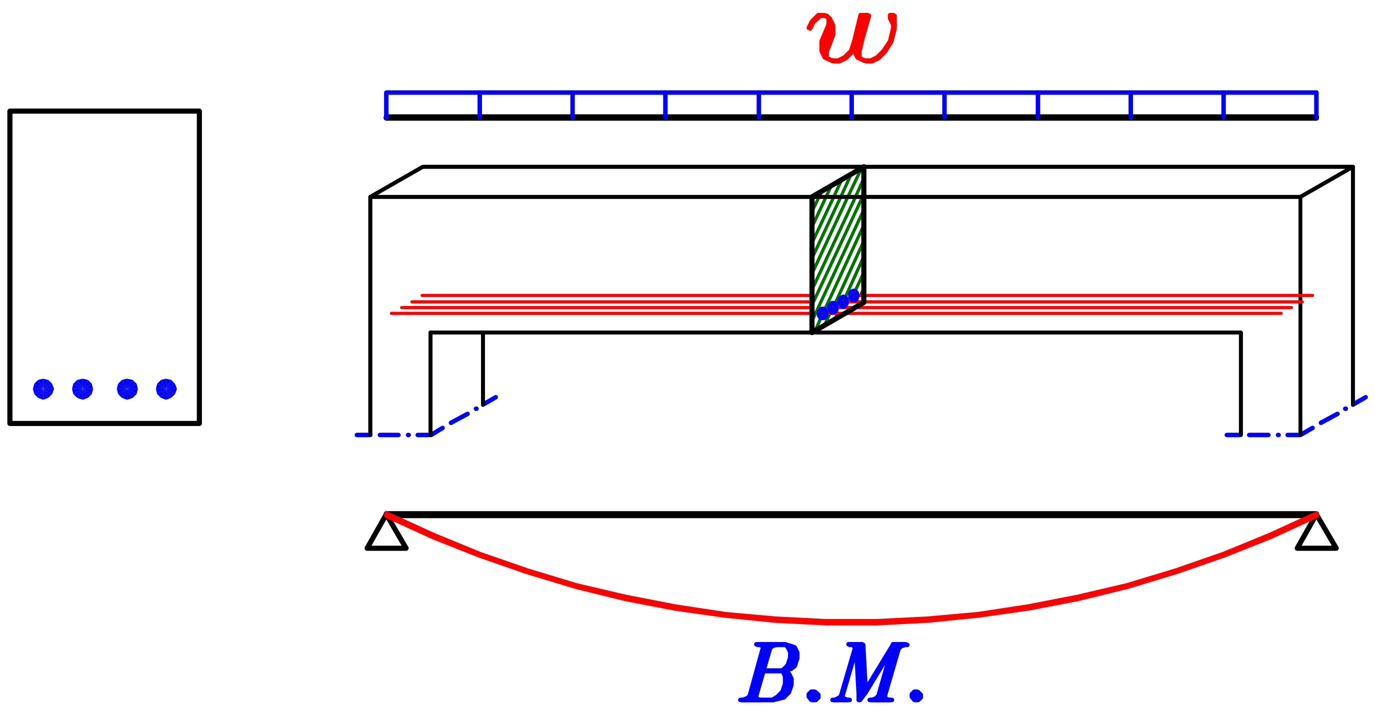
Behavior of Beams
under Bending Moment only

خواص الكمرات تحت تأثير عزوم الانحناء فقط

نسألكم الدعاء

Behavior of Beams Table of Contents.

Introduction.



كثيرا ما نحتاج لحساب قوه تحمل قطاعات الكمره للعزوم المؤثره عليها .

أى نحتاج لحساب أكبر عزوم يستطيع القطاع تحملها فى الحالات المختلفه مثل :

1- ($M_{cr.}$) Cracking Moment

($M_{cr.}$) هو العزم الذى تبدأ عنده الخرسانه من جهه الشد فى التشرخ .

2- (M_w) Working Moment

(M_w) هو أكبر عزم مسموح به للكمرات الشغاله و الذى يجعلها Just safe

و اذا عرض القطاع لعزم أكبر من (M_w) يكون unsafe فى طريقه W.S.D.M.

Working Stress Design Method

3- (M_{ult}) Ultimae Moment.

(M_{ult}) هو أكبر عزم يتحمله القطاع و اذا تعرض القطاع لعزم أكبر ينهار.

4- ($M_{U.L.}$) Ultimae Limits Moment.

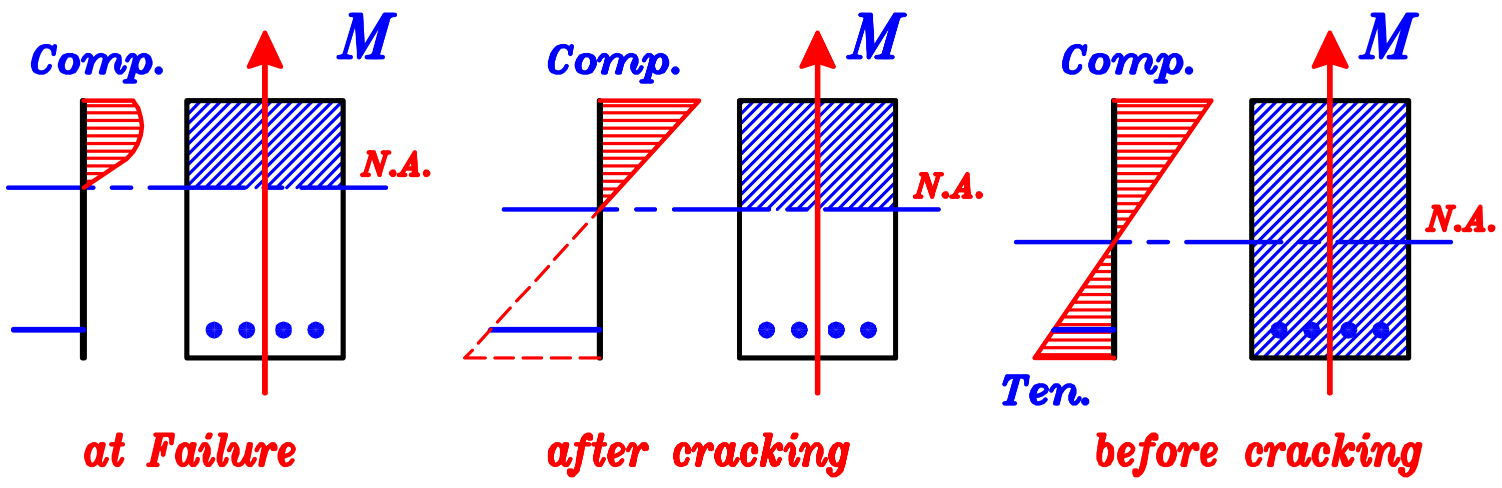
($M_{U.L.}$) هو أكبر عزم مسموح به للكمرات الشغاله و الذى يجعلها Just safe

و اذا عرض القطاع لعزم أكبر من ($M_{U.L.}$) يكون unsafe فى طريقه U.L.D.M.

Ultimae Limits Design Method

و لكى نستطيع أن نحسب العزوم التى يتحملها القطاع .
يجب أولا دراسه بعض خواص الخرسانه و الحديد المستخدمين فى القطاع .
و أيضا دراسه بعض الخواص الهندسيه للقطاع و معرفه بعض المبادئ الاساسيه للعناصر الانشائيه .

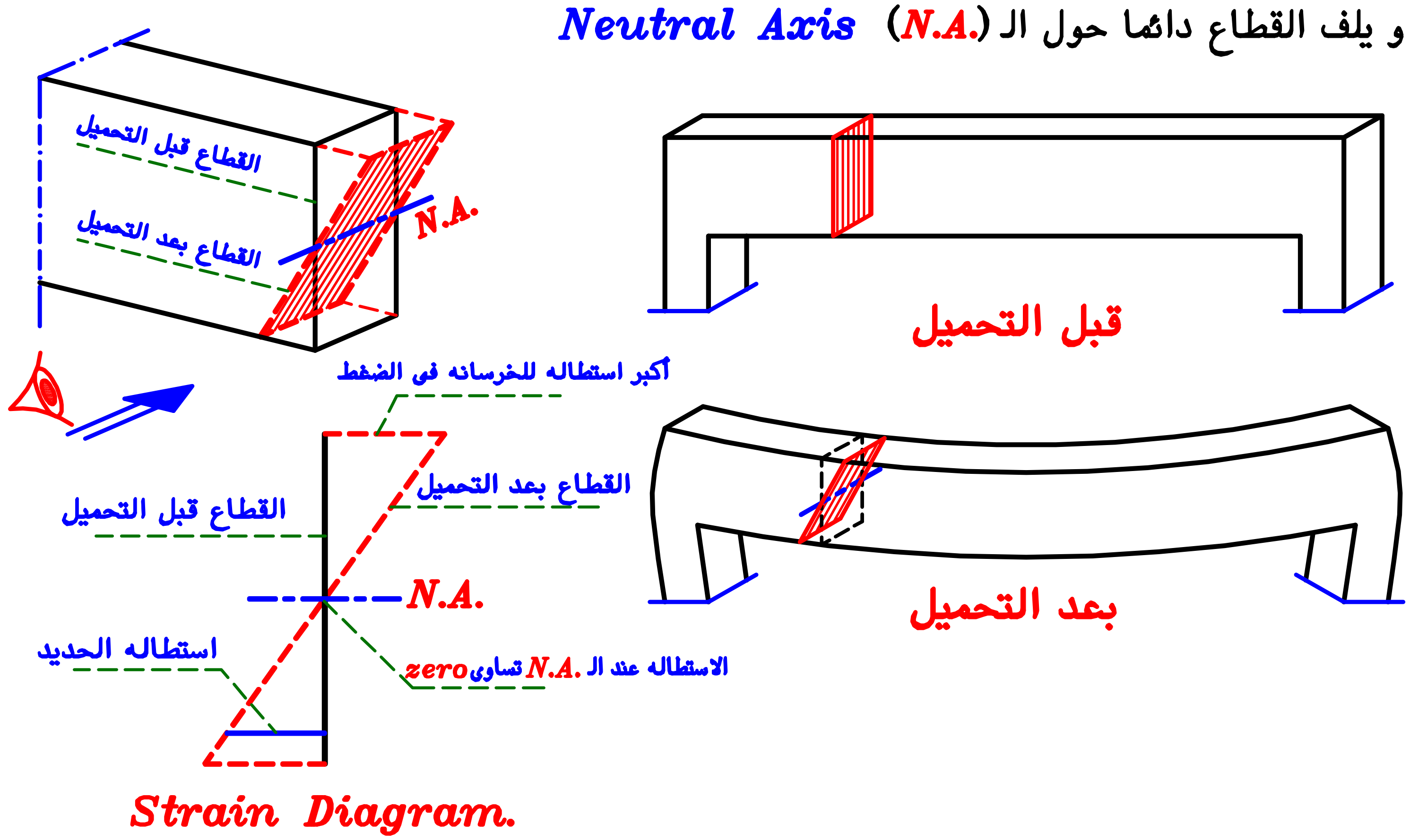
Stress Diagram For section under Bending Moment only.



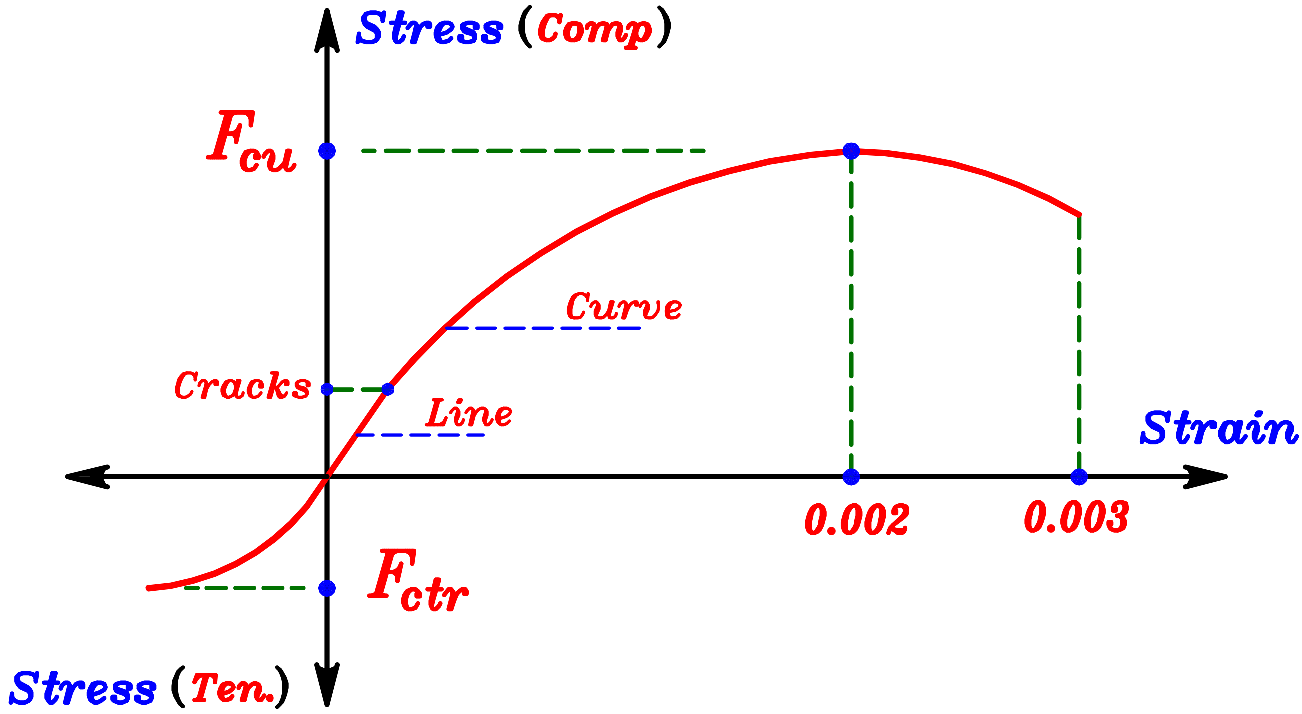
Strain Diagram For sections.

Elastic Theory.

هى نظريه تعتمد على أن شكل القطاع المستوى قبل تحميل الكمره يظل مستوى بعد التحميل .
و يلف القطاع دائما حول الـ (N.A.) Neutral Axis



Stress – Strain Curve For Concrete.



F_{cu} هى أكبر اجهاد تتحمله الخرسانه فى الضغط

و تتوقف قيمتها على تصميم الخلطه الخرسانيه .

رتبه الخرسانه							
F_{cu} ($N \backslash mm^2$)	18	20	25	30	35	40	45

F_{ctr} هى أكبر اجهاد تتحمله الخرسانه فى الشد .

واذا زاد اجهاد الشد فى الخرسانه عن هذه القيمه تحدث شروخ فى الخرسانه.

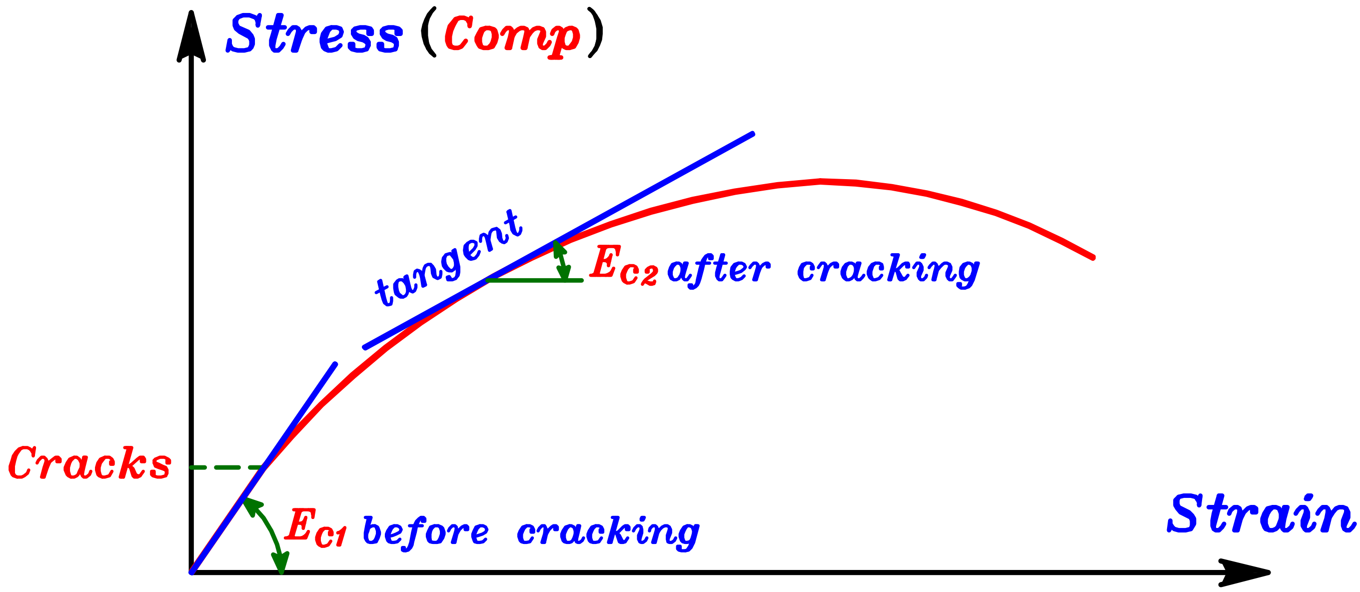
$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}} \quad N \backslash mm^2$$

F_{ctr} (Concrete Tension Rupture)

Modules of elasticity of Concrete. (E_c)

معاير مرونه الخرسانه

$$E = \frac{stress}{strain}$$



$$\boxed{E_{c_1} = 4400\sqrt{F_{cu}}}\quad N\backslash mm^2$$

E_{c_1} = modules of elasticity of concrete before craking.

و هو عباره عن ميل خط ال stress–strain curve قبل التشرخ .

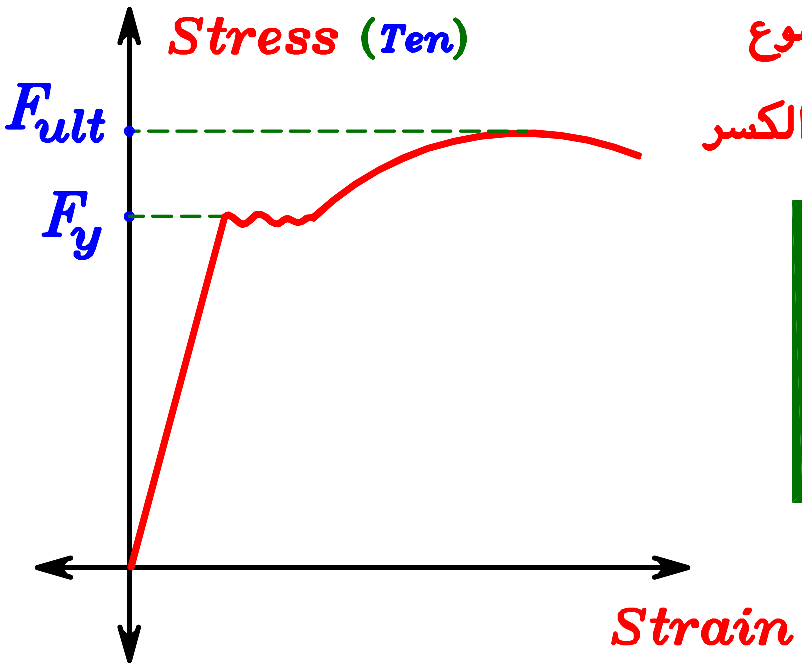
E_{c_2} = modules of elasticity of concrete after craking.

و هو عباره عن ميل المماس لل curve عند أى نقطه بعد التشرخ .

و لا يوجد لها معادله هى فقط ميل مماس ال curve عند النقطه المحسوب عندها E

$$\boxed{E_{c_2} < E_{c_1}}$$

Stress−Strain Curve For Steel in Tension.



اجهاد الخضوع yield stress (F_y)

اجهاد الكسر ultimate Stress (F_{ult})

انواع الحديد المستخدم فى مصر
Mild Steel 240\350
High strength Steel. 360\520
High strength Steel. 400\600

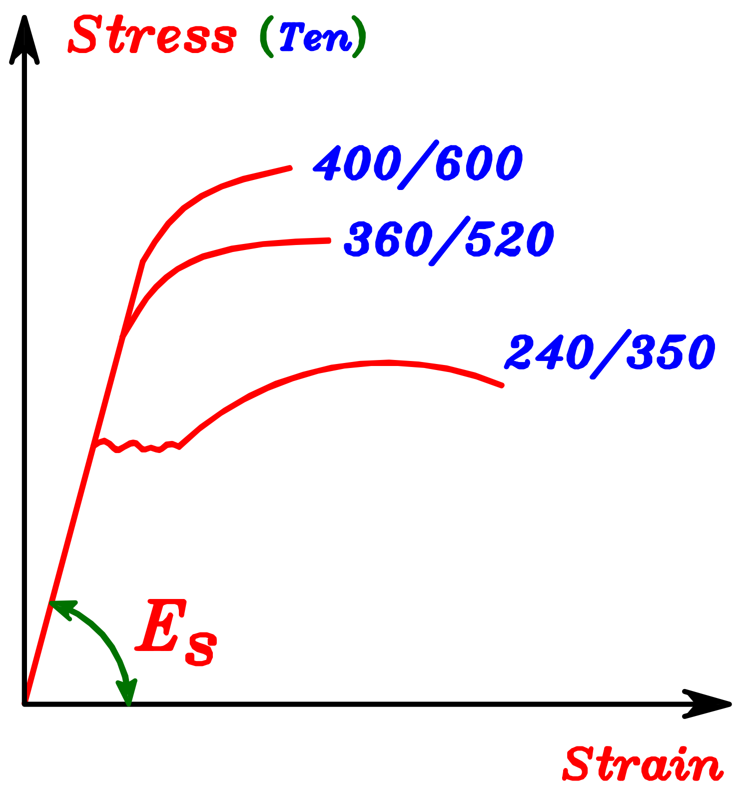
Modules of elasticity of Steel. (E_s)

معاير مرونه الحديد

For all types of steel

$$\boxed{E_s = 2 * 10^5} \quad N\backslash mm^2$$

E_s (Young's Modules)



Modular Ratio (n)

$$\boxed{n = \frac{E_s}{E_c}}$$

E_s = constant = $2 * 10^5$ N\mm²

$E_{c_1} = 4400 \sqrt{F_{cu}}$ N\mm² --- before cracking

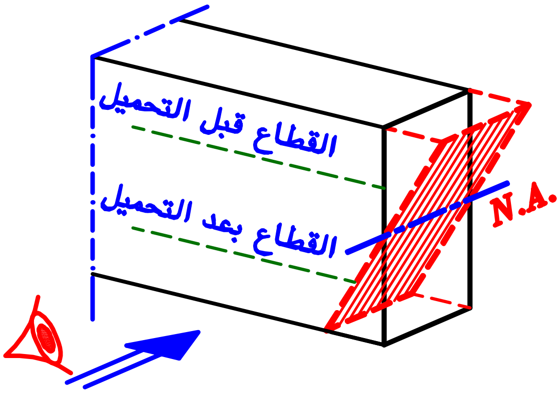
$E_{c_2} < E_{c_1}$ --------------- after cracking

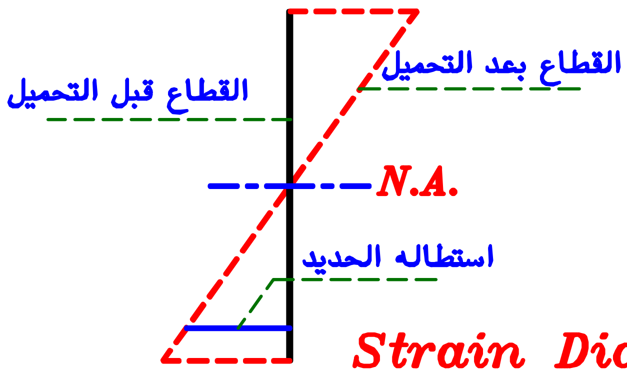
Before cracking $$n = \frac{E_s}{E_{c1}} = \frac{2 * 10^5}{4400\sqrt{F_{cu}}} \simeq 10$$

After cracking $$n = \frac{E_s}{E_{c2}} \simeq 15$$

$$n = \frac{E_s}{E_c} = \frac{(\text{stress} \backslash \cancel{\text{strain}})\ \text{steel.}}{(\text{stress} \backslash \cancel{\text{strain}})\ \text{conc.}} = 10$$

و معناه إنه إذا حدث للحديد نفس الإستطاله الحادثه للخرسانه سوف يكون على الحديد إجهادات (n) مره الإجهادات الواقعه على الخرسانه.



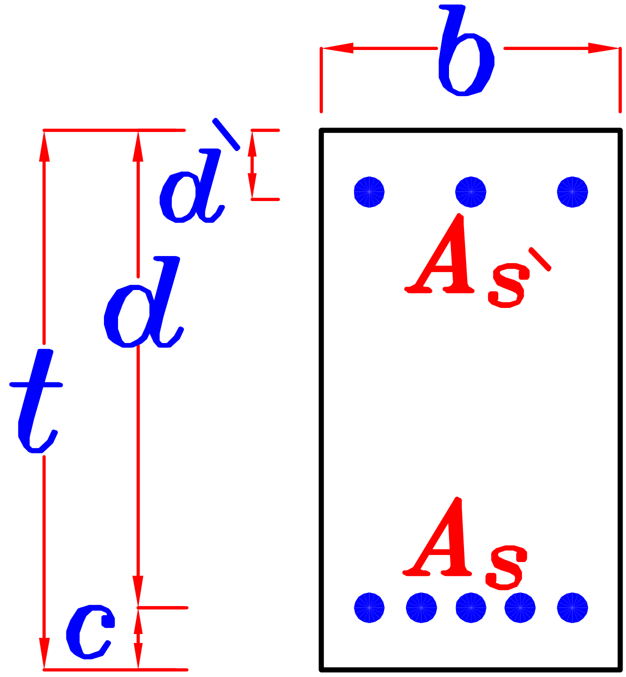


Strain Diagram.

و لأنة من المفترض أن القطاع المستوى قبل التحميل يظل مستوى بعد التحميل فهذا معناه أن الإستطاله *Strain* الحادثه فى الحديد هى نفس الإستطاله الحادثه فى الخرسانه الملاصقه للحديد.

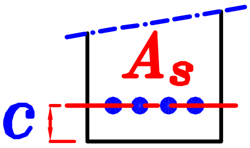
و هذا معناه أن الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له.

Important Symbols. رموز هامه

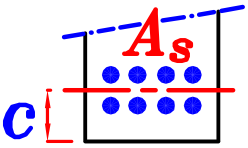


Width عرض القطاع $= b$

Depth عمق القطاع $= t$



$c \simeq 50\,mm$



$c \simeq 75\,mm$

Tension cover غطاء حديد الشد $= c$

و يحسب من C.G. أسياخ الحديد

$\boxed{d = t - c}$

Effective depth العمق الفعلى $= d$

$\boxed{d` \simeq 50\,mm}$

Compression cover غطاء حديد الضغط $= d`$

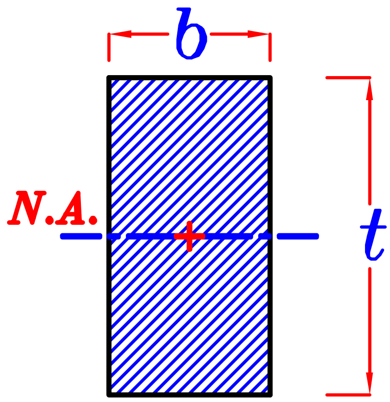
Area of tension steel مساحه حديد الشد $= A_s$

Area of compression steel مساحه حديد الضغط $= A_s`$

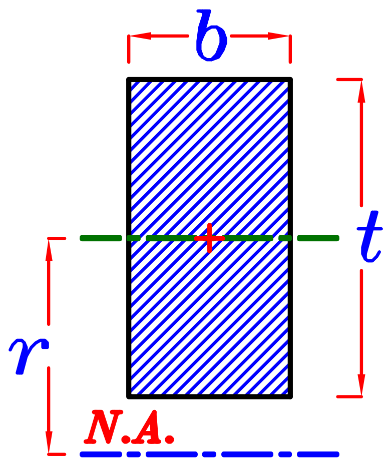
Moment of Inertia.

ملحوظه دائما نحسب ال **Inertia** **(I)** للقطاع حول ال **(N.A.)** **Neutral Axis**

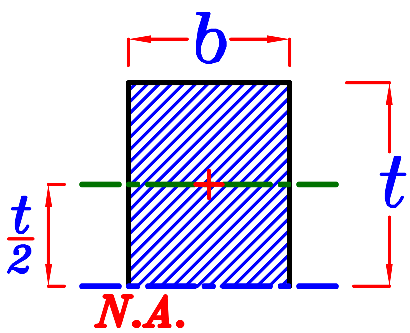
$$I = \frac{b\,t^3}{12}$$



$$I = \frac{b\,t^3}{12} + (b\,t)\,(r)^2$$

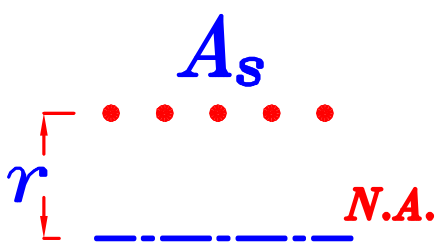


$$I = \frac{b\,t^3}{3}$$



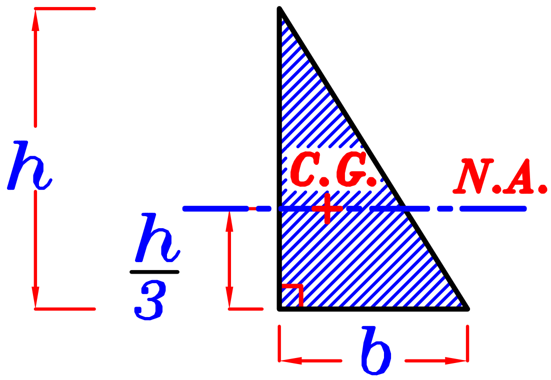
For Steel Bars.

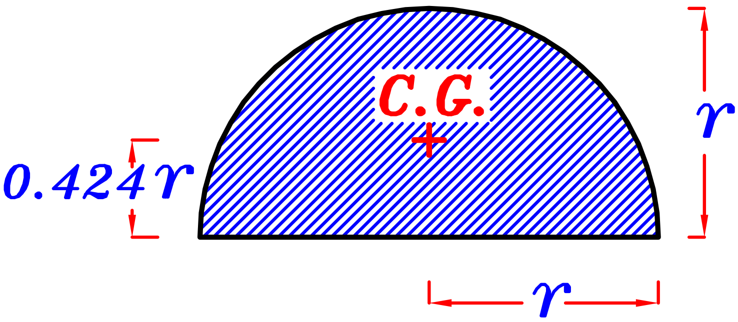
$$I = A_s\,(r)^2$$

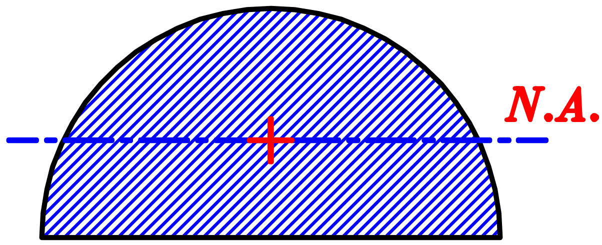


Special Cases.

$$I_x = \frac{b\,h^3}{36}$$

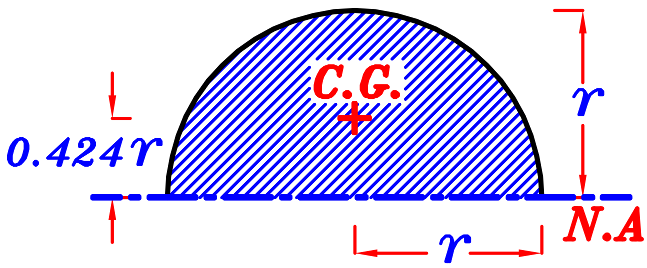


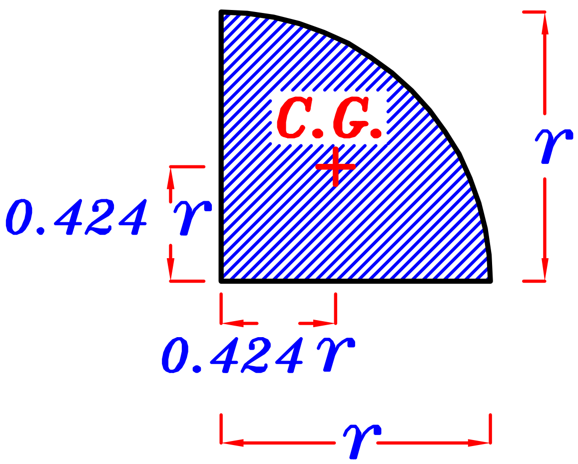


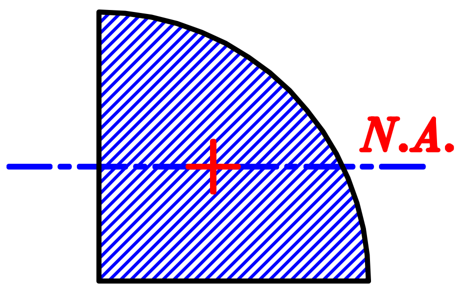


$$I = 0.11\,r^4$$

$$I = 0.11\,r^4 + \left(\frac{\pi r^2}{2}\right)\left(0.424\,r\right)^2$$







$$I_x = 0.055\,r^4$$

Virtual Section. القطاع التخيلى

لحساب ال Inertia (I) لقطاع بالقوانين السابقه

يجب أن يكون القطاع متجانس (homogeneous section) أى يتكون من ماده واحده فقط

أما اذا كان القطاع غير متجانس (heterogeneous section) أى يتكون من أكثر من ماده

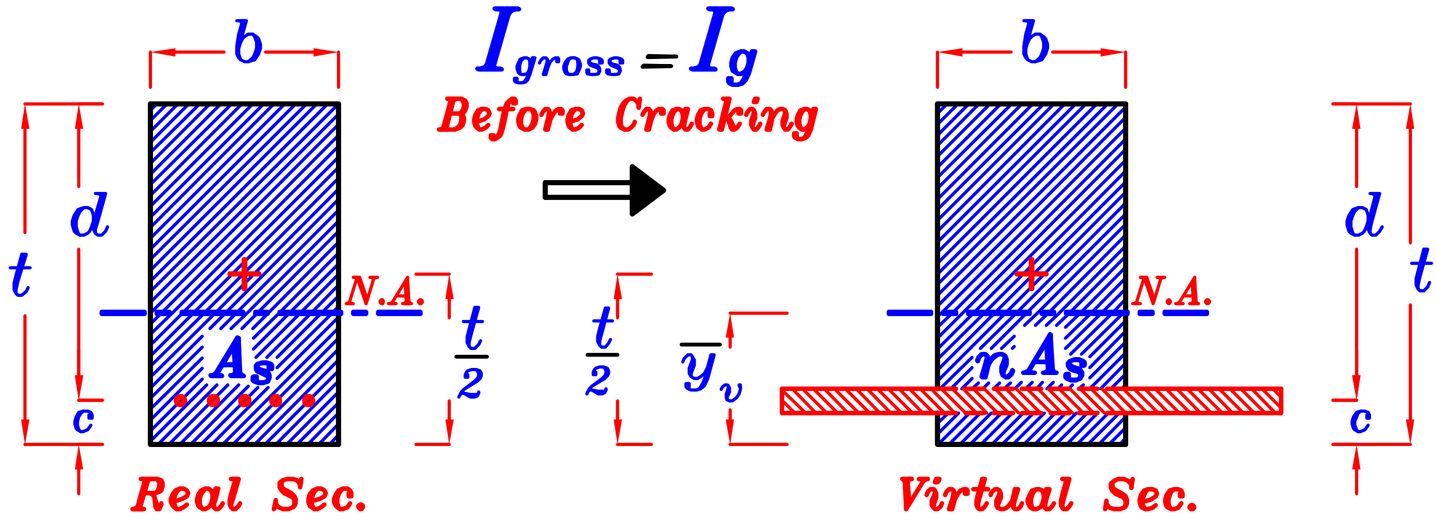
فيجب عمل حل تخيلى و هو بأفتراض أن القطاع يتكون من ماده واحده فقط و هى الخرسانه

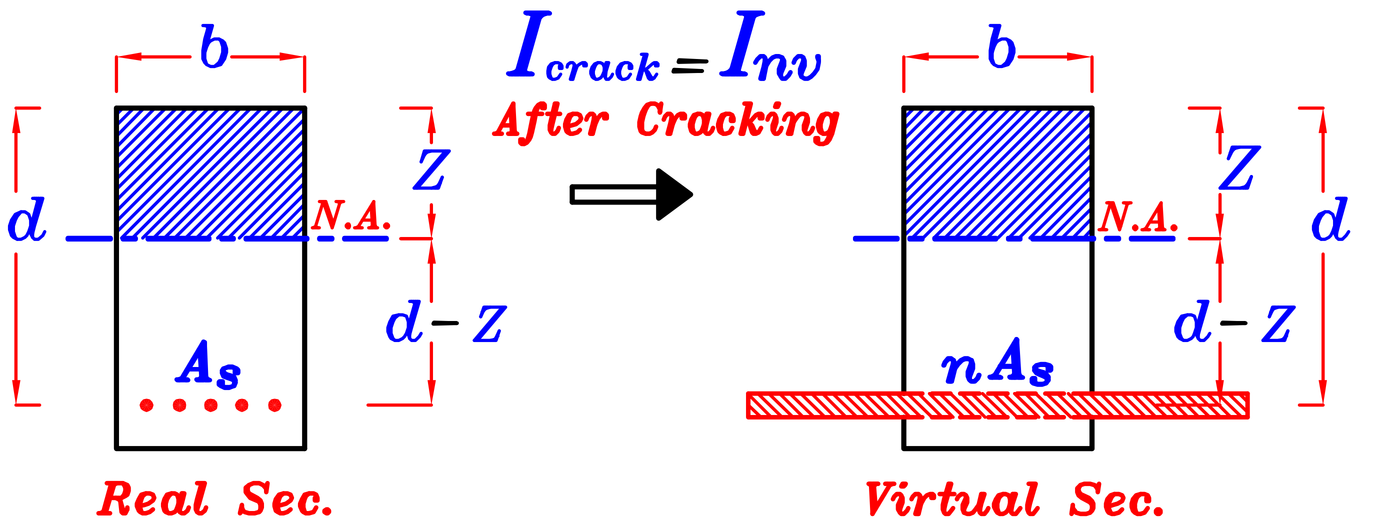
و لان الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له

فمن الممكن ان نتخيل انه بدل الحديد الموجود فى القطاع يوجد مكانه خرسانه مساحتها (n) مره مساحه

الحديد و موضوعه فى نفس المكان

بهذه الطريقه نستطيع حساب ال (I) للقطاع التخيلى فتكون هى نفس ال (I) للقطاع الحقيقى .

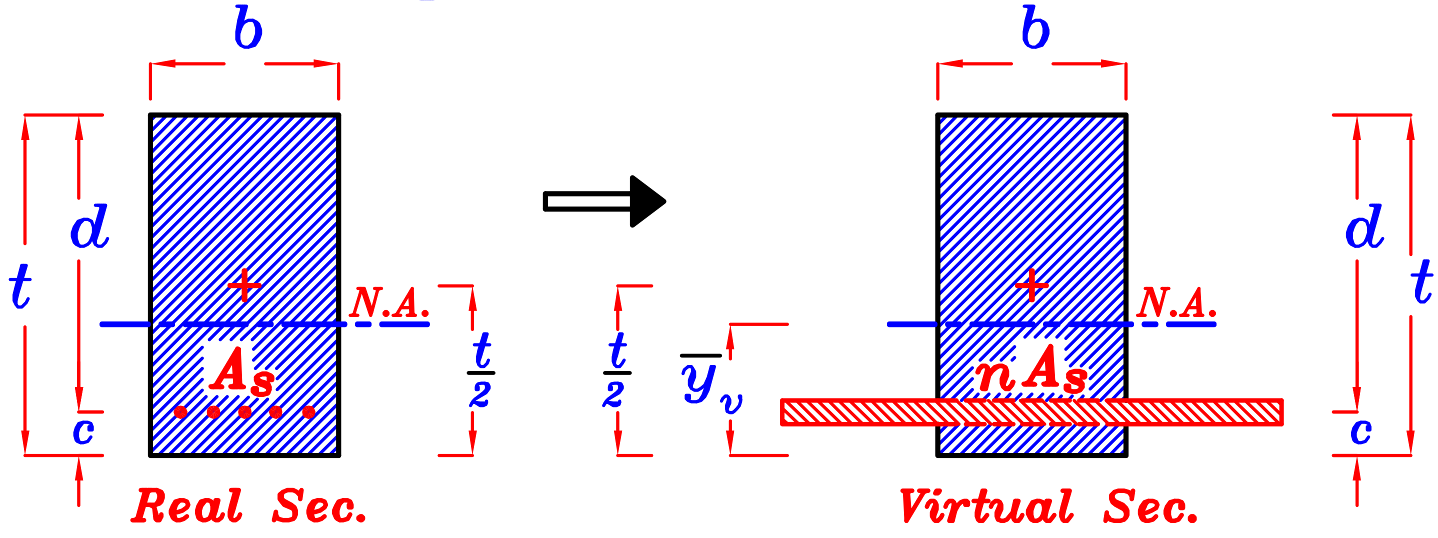




ملحوظه دائما نحسب ال Inertia (I) للقطاع حول ال (N.A.) Neutral Axis

Before cracking. I_g

without compression steel $A_s`$



n (before cracking) $\simeq 10$

c = cover From tension steel $\simeq (40 \longrightarrow 50)$ mm.

d = distance From tension steel to max compression Fibers.

قبل أن تتشرخ الخرسانه يكون مكان ال (N.A.) عند ال (C.G.) للقطاع لذا نحدد $\bar{y}$ قبل حساب ال (I)

$$A_c = b * t - A_s$$

$$A_v = A_c + n A_s = b * t - A_s + n A_s = b * t + (n-1) A_s$$

$$\boxed{A_v = b * t + (n-1) A_s}$$

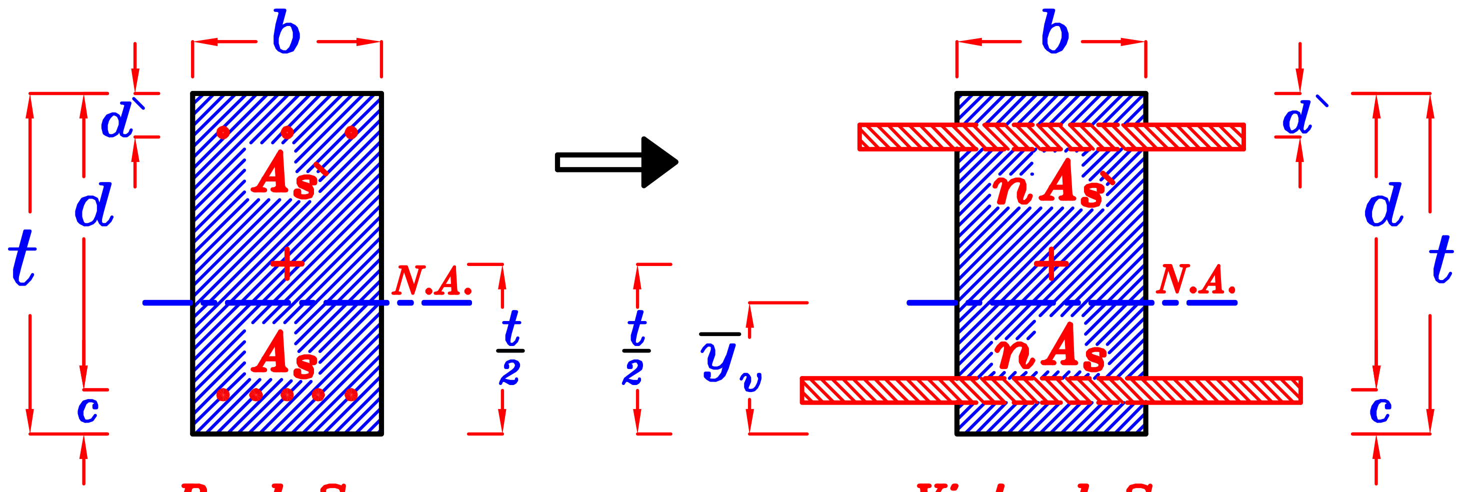
$\bar{y}_t$ = C.G. of virtual Sec. From Tension side.

$$\boxed{\bar{y}_t = \frac{b * t * \frac{t}{2} + (n-1) A_s * c}{A_v}}$$

I_g = moment of inertia about N.A. For virtual Sec.

$$\boxed{I_g = \frac{b * t^3}{12} + b * t \left(\frac{t}{2} - \bar{y}_v\right)^2 + (n-1) A_s \left(\bar{y}_v - c\right)^2}$$

with compression steel $A_s`$



Real Sec. Virtual Sec.

$d`$ = distance From Compassion steel to max compression Fibers.

IF $A_{s`} < 0.2\ A_s$
$\therefore$ We can neglect $A_{s`}$

$$A_c = b * t - A_s - A_{s`}$$

$$A_v = A_c + nA_s + nA_{s`}$$

$$= b * t - A_s - A_{s`} + nA_s + nA_{s`}$$

$$\boxed{A_v = b * t + (n-1)A_s + (n-1)A_{s`}}$$

$\bar{y}_t$ = C.G. of virtual Sec. From Tension side.

$$\boxed{\bar{y}_t = \frac{b * t * \frac{t}{2} + (n-1)A_s * c + (n-1)A_{s`} * (t - d`)}{A_v}}$$

I_g = moment of inertia about N.A. For virtual Sec.

$$\boxed{I_g = \frac{b * t^3}{12} + b * t\ \left(\frac{t}{2} - \bar{y}_v\right)^2 + (n-1)A_s\ (\bar{y}_v - c)^2 + (n-1)A_{s`}\left[(t - d`) - \bar{y}_v\right]^2}$$

After cracking. I_{nv}

عند تشرخ الخرسانه من جهه الشد يتحرك الـ (N.A.) جهه الضغط قليلا ليوازن القطاع من جديد
و بالتالى لن يكون الـ (N.A.) عند الـ (C.G.) القديمه للقطاع
و لكى نستطيع أن نحدد مكان الـ (N.A.) الجديد نحدده عن طريق الاتزان
أى يجب أن يكون مجموع ضرب المساحات فى بعد مركزها عن الـ (N.A.) أسفل الـ (N.A.)
تساوى مجموع ضرب المساحات فى بعد مركزها عن الـ (N.A.) أعلى الـ (N.A.)

Area * distance = $S_{nv.}$ (First Moment of Area)

$$\boxed{S_{nv.\,\text{above (N.A.)}} = S_{nv.\,\text{under (N.A.)}}}$$

nv. means about (N.A.) For Virtiual section

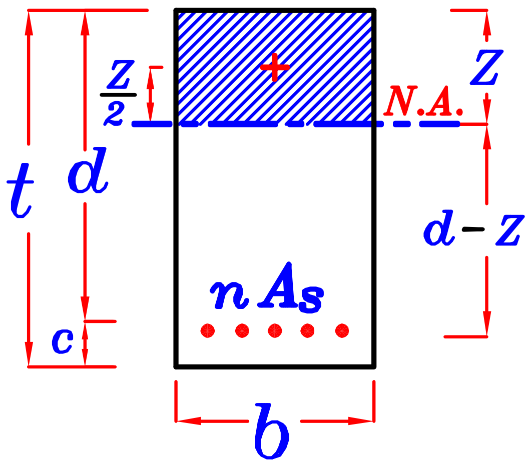
① For R-Sec.

without compression steel $A_s`$

n (after cracking) $\simeq 15$

Get Z (From Comp. side)

by taking $\boxed{S_{nv.\,\text{above (N.A.)}} = S_{nv.\,\text{under (N.A.)}}}$



$$\boxed{b\,(z)\left(\frac{Z}{2}\right) = n\,A_s\,(d-Z)}$$

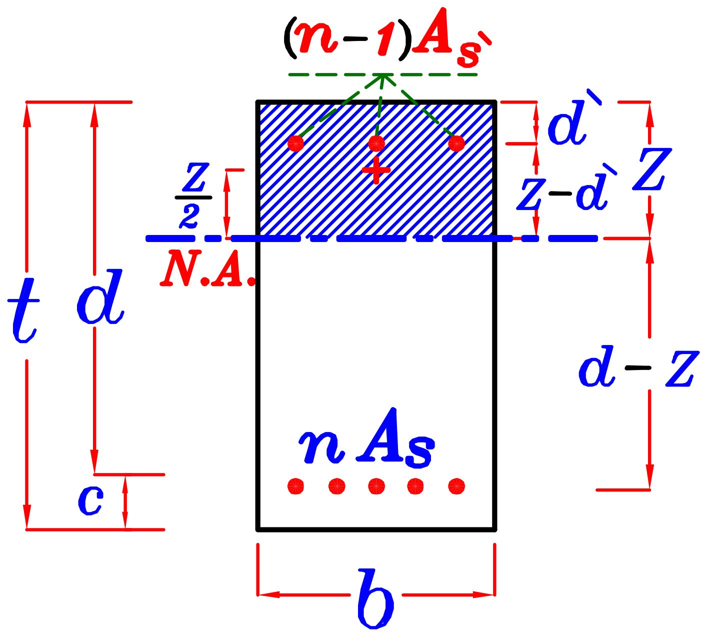
Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{bZ^3}{3} + n\,A_s\,(d-Z)^2}$$

with compression steel A_{s}' $\boxed{IF\ A_{s}' > 0.2\ A_s}$

n (after cracking) $\simeq 15$

Get Z (From Comp. side)



$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

$$\boxed{b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_{s}'\,(Z-d') = n\,A_s\,(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{bZ^3}{3} + (n-1)\,A_{s}'\,(Z-d')^2 + n\,A_s\,(d-Z)^2}$$

② For T-Sec. or L-Sec.

(Tension Steel only)

No Compresion steel in T-sec. & L-sec.

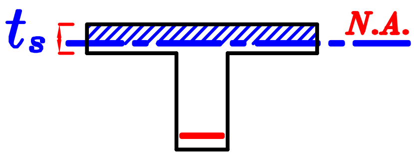
To know IF the N.A. above or under the Flange.

Assume that the N.A. is exactly at the Flange.

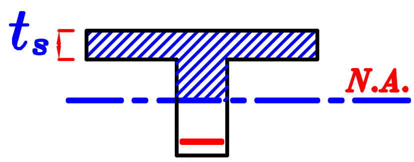
Calculate (First Moment of Area) $S_{nv.}$

above and under the Flange.

IF $S_{nv.}(above) > S_{nv.}(under)$ $\therefore Z < t_s$ →



IF $S_{nv.}(under) > S_{nv.}(above)$ $\therefore Z > t_s$ →

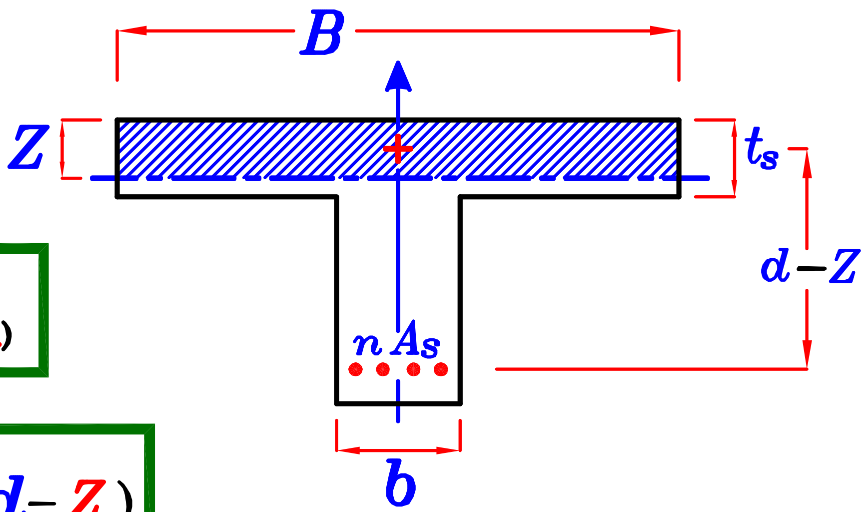


ⓐ IF $S_{nv.}(above) > S_{nv.}(under)$ $\therefore Z < t_s$

$\therefore$ The sec. will act the same as R-sec. but with width B

n (after cracking) $\simeq 15$

Get Z (From Comp. side)



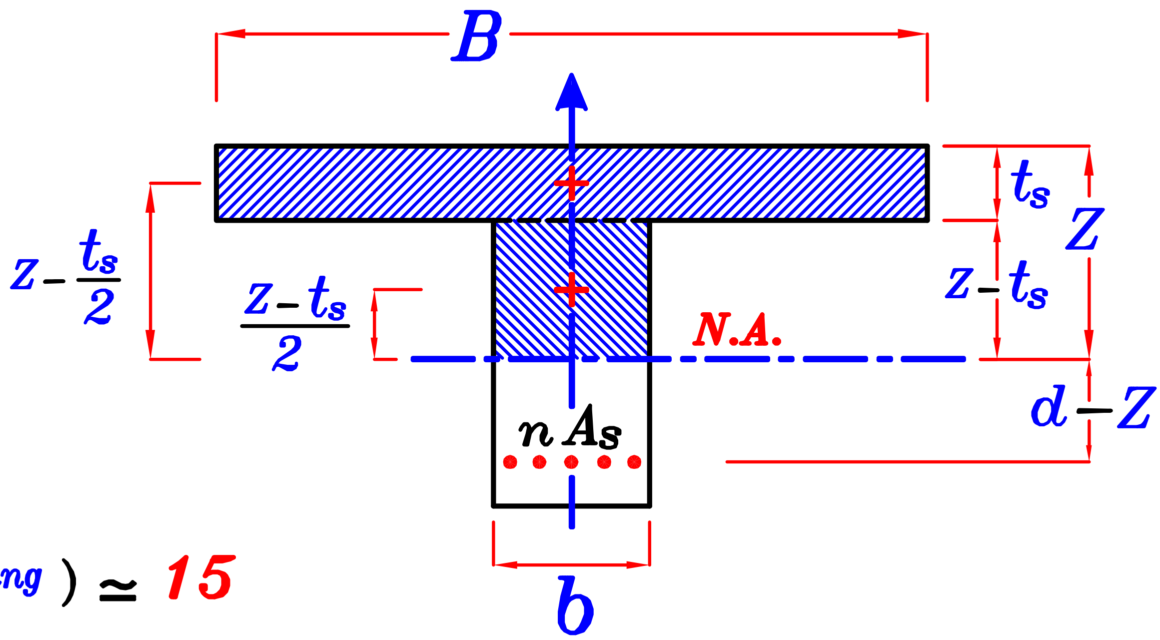
$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

$$\boxed{B(Z)\left(\frac{Z}{2}\right) = n\,A_s(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{B\,Z^3}{3} + n\,A_s(d-Z)^2}$$

(b) IF $S_{nv.}(above) < S_{nv.}(under)$ $\therefore Z > t_F$



n (after cracking) $\simeq 15$

Get Z (From Comp. side)

$$\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$$

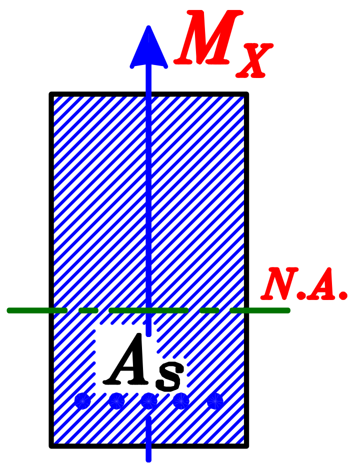
$$\boxed{B(t_s)\left(Z-\frac{t_s}{2}\right) + b\,(Z-t_s)\left(\frac{Z-t_s}{2}\right) = n\,A_s\,(d-Z)}$$

Get $I_{cr.} = I_{nv}$ (moment of inertia For cracked section)

$$\boxed{I_{nv} = I_{cr.} = \frac{B\,t_s^3}{12} + B\,(t_s)\left(Z-\frac{t_s}{2}\right)^2 + \frac{b\,(Z-t_s)^3}{3} + n\,A_s\,(d-Z)^2}$$

لحساب ال Normal stress على الخرسانه فى أى قطاع نستخدم معادله :

$$F = -\frac{N}{A} \pm \frac{M_Y\,x}{I_Y} \pm \frac{M_X\,y}{I_X}$$



و لاننا نتحدث على كمرات فلا يوجد عليها قوى محوريه $N = Zero$

$$\therefore \quad F = \pm \frac{M_Y\,x}{I_Y} \pm \frac{M_X\,y}{I_X}$$

و لاننا نتحدث على أوزان فقط و لا نتحدث عن قوى افقيه فبالتالى يكون العزم رأسى فقط $M_Y = Zero$

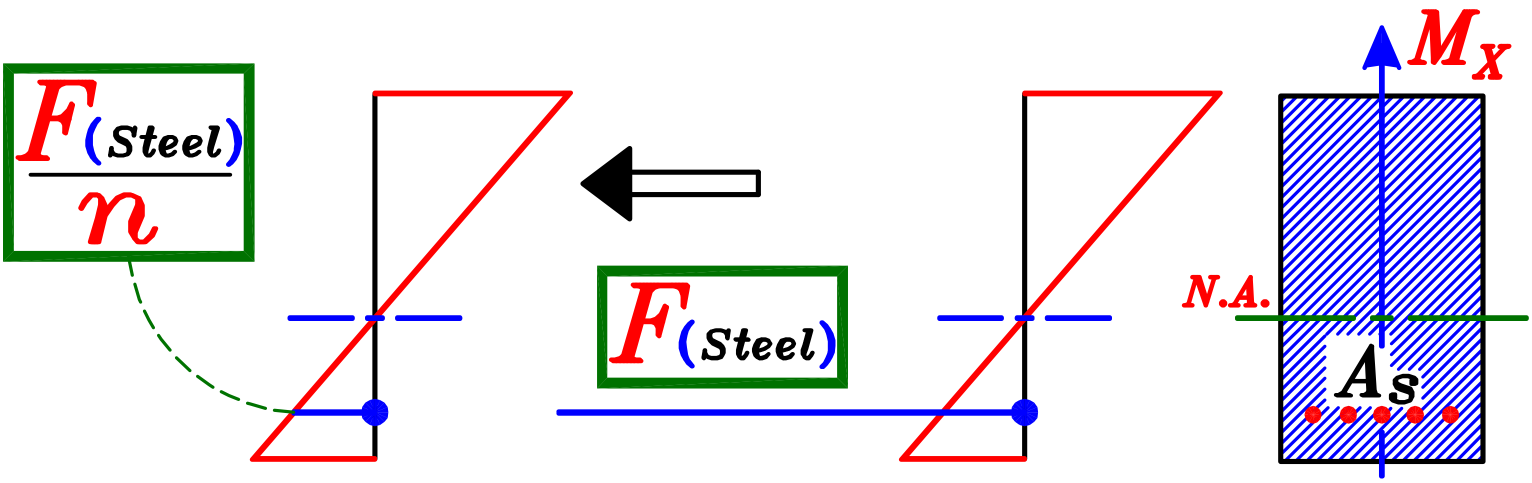
$$\therefore \quad \boxed{F = \pm \frac{M_X\,y}{I_X}}$$

Normal stress
على الخرسانه

و لان الاجهادات الواقعه على الحديد تساوى (n) مره الاجهادات الواقعه على الخرسانه الملاصقه له.

$$\therefore \quad \boxed{F = \pm\, n * \frac{M_X\,y}{I_X}}$$

Normal stress
على الحديد



$$\boxed{F = \frac{M\,y}{I}}$$ (Concrete)

$$\boxed{F = n\,\frac{M\,y}{I}}$$ (Steel)

Where :

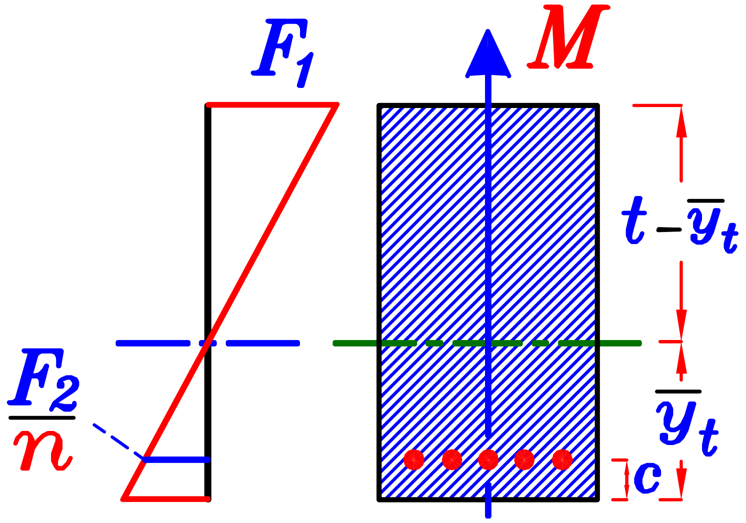
y هى المسافه من النقطه المحسوب عندها ال stress حتى ال N.A.

I هى ال moment of Inertia للقطاع الشغال حول ال N.A.

و تساوى $I = I_g$ للقطاع قبل التشرخ before cracking

و تساوى $I = I_{nv}$ للقطاع بعد التشرخ after cracking

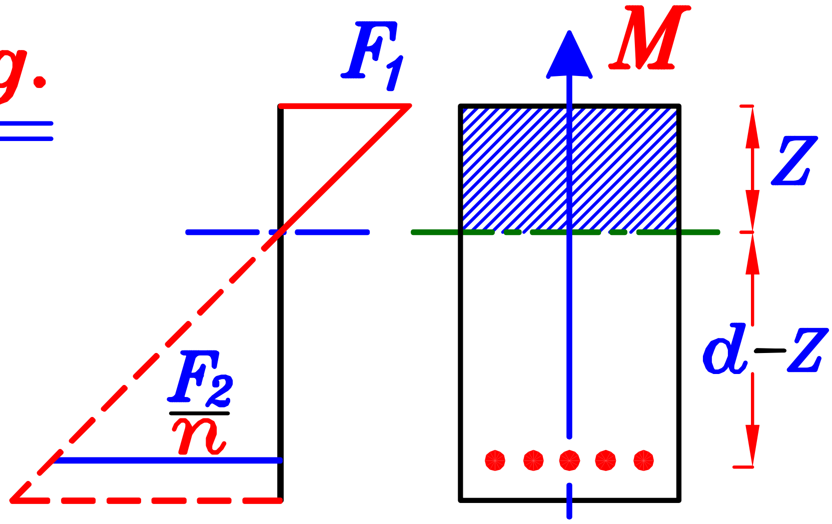
Before Cracking.



$$F_{1\,(Concrete)} = \frac{M * y}{I} = \frac{M * (t - \bar{y}_t)}{I_g}$$

$$F_{2\,(Steel)} = n\,\frac{M * y}{I} = 10 * \frac{M * (\bar{y}_t - c)}{I_g}$$

Before Cracking.



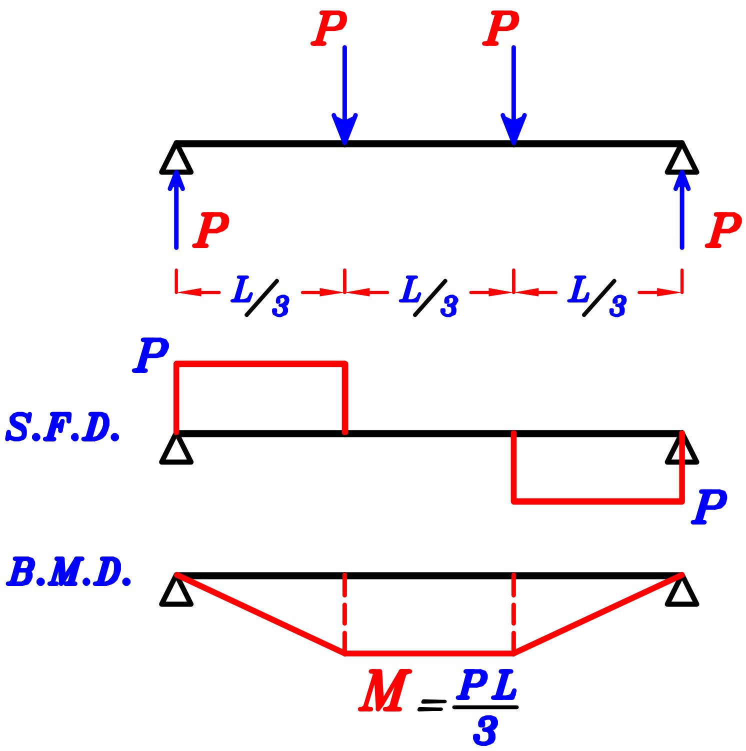
$$F_{1\,(Concrete)} = \frac{M * y}{I} = \frac{M * Z}{I_{nv}}$$

$$F_{2\,(Steel)} = n\,\frac{M * y}{I} = 15 * \frac{M * (d - Z)}{I_{nv}}$$

Stages of Beams under Variable Bending Moment.

لدراسه خواص الكمره تحت تأثير حالات التحميل المختلفه .

ندرس كمره (*Simply Supported*) كما هو مبين فى الشكل (مع اهمال وزنها o.w.) .



حيث يكون الثلث الأوسط من الكمره يوجد عليه $B.M.$ فقط .

و لا يوجد علية $S.F.$ و هذا هو الجزء هو الذى سندرسه .

بزياده مقدار القوى P يزداد مقدار العزم الواقع على على الكمره $M = \frac{PL}{3}$

و بدراسه الكمره مع زياده الحمل نجد أنها تمر بثلاث مراحل :

1 – 0.0 $\longrightarrow$ Cracking.

2 – Cracking $\longrightarrow$ Working.

3 – Working $\longrightarrow$ Ultimate.

1– Cracking Stage.

$M = 0.0 \longrightarrow M_{cr.}$

$P_{cr.}$ هو الحمل الذى يحدث عنده أول شرخ فى الكمره من جهه الشد Tension Side

$$M_{cr.} = (P_{cr.} * L) \backslash 3$$

2– Working Stage.

$M_{cr.} \longrightarrow M_w$

P_w هو الحمل الذى يصل عنده الاجهاد على أى من الحديد أو الخرسانه الى $F_{allowable}$

$$M_w = (P_w * L) \backslash 3$$

3– Ultimate Stage.

$M_w \longrightarrow M_{ult.}$

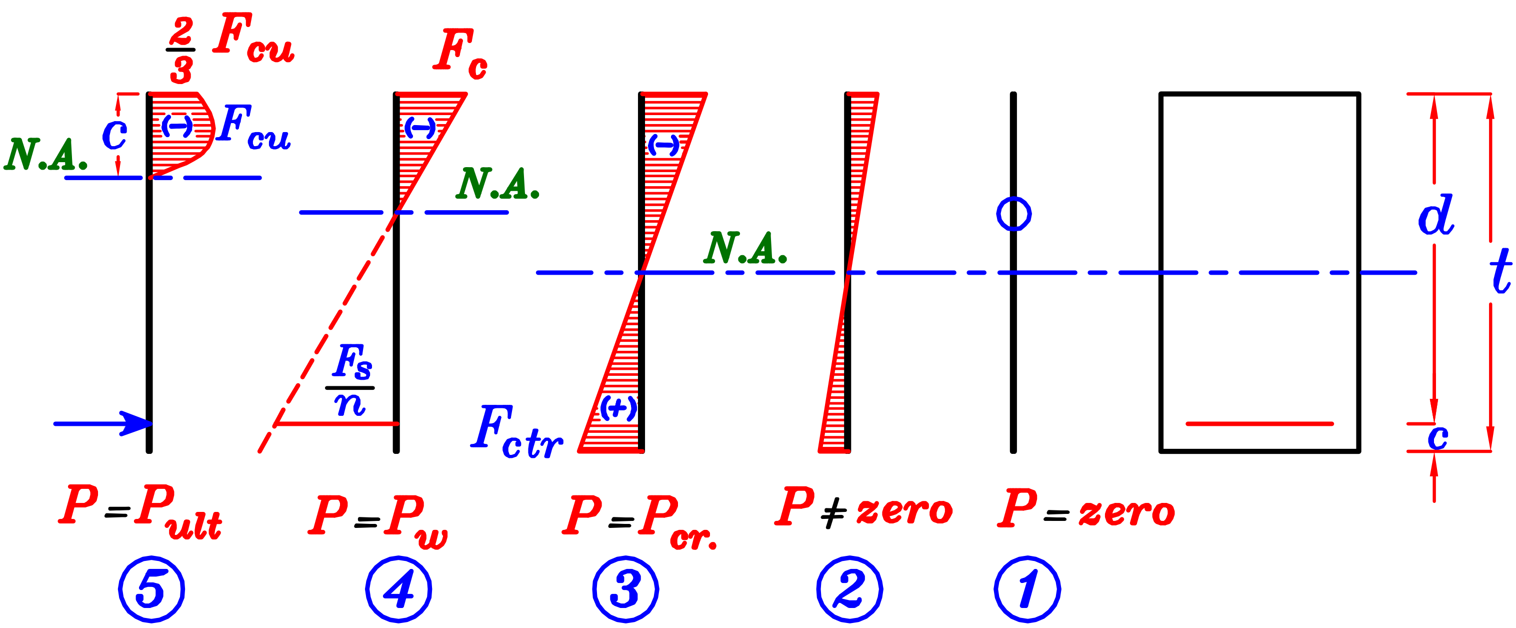
$P_{ult.}$ هو الحمل الذى يحدث عنده انهيار للكمره أى يصل الاجهاد على الخرسانه فى الضغط الى F_{cu}

أو يصل الاجهاد على الحديد فى الشد الى F_y

$$M_{ult.} = (P_{ult.} * L) \backslash 3$$

Normal Stresses Diagram

For beams subjected to Bending Moment only.



١ـ قبل التحميل يكون الـ normal stress = Zero

٢ـ فى بدايه التحميل يحدث شد فى السطح السفلى و ضغط فى السطح العلوى

٣ـ مع زياده الحمل يزيد الـ normal stress حتى يصل فى منطقه الشد الى F_{ctr}

و عند هذه اللحظه يسمى الحمل $P_{cr.}$ و يسمى العزم $M_{cr.}$

٤ـ مع زياده الحمل تظهر شروخ فى الخرسانه فى منطقه الشد

(الجزء المتشرخ من الخرسانه لا يؤخذ فى الحساب أى كأنه غير موجود)

و مع زياده الحمل يصل الاجهاد فى الخرسانه فى منطقه الضغط الى F_c Allowable stresses

أو يصل الاجهاد فى الحديد فى منطقه الشد الى F_s Allowable stresses

و عند هذه اللحظه يسمى الحمل P_w و يسمى العزم M_w

٥ـ مع زياده الحمل يزداد الضغط على الخرسانه و يحدث تغير غير منتظم فى الاجهادات

non Linear stresses على الخرسانه

حتى يصل الاجهاد فى الخرسانه فى منطقه الضغط الى F_{cu}

أو يصل الاجهاد فى الحديد فى منطقه الشد الى F_y

و تبدأ الكمره فى الانهيار و عند هذه اللحظه يسمى الحمل P_{ult} و يسمى العزم M_{ult}

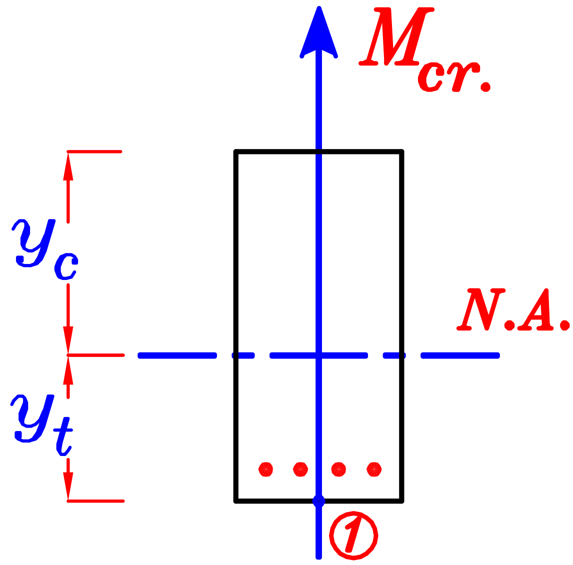
Cracking Moment ($M_{cr.}$)

هو قيمه العزم الذى يؤدى الى حدوث أول شرخ فى الخرسانه من جهه الشد
و عنده يصل الإجهاد فى الخرسانه فى منطقه الشد الى F_{ctr}

$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}} \quad N\backslash mm^2$$

Cracking Tensile stress. (Concrete Tension Rupture)

$$\because\ F = \frac{M * y}{I} \Longrightarrow M = \frac{F * I}{y}$$



at cracking

F at point ① $= F_{ctr}$

$\therefore$ Moment at this case $= M_{cr.}$

$$\boxed{M_{cr.} = \frac{F_{ctr} * I_g}{y_t}}$$

$M_{cr.}$ = Cracking moment

I_g = Moment of Inertia around N.A.
(For virtual sec.)

y_t = Distance between N.A. to extreme tension Fibers.
(For virtual sec.)

عندما يكون شكل المقطاع معطى و مطلوب $M_{cr.}$

أى يطلب قيمه العزم الذى سوف يسبب التشرخ للخرسانه فى منطقه الشد .

تكون خطواط الحل كالاتى :

١ - نحسب n $\quad n = \frac{E_s}{E_{c1}} = \frac{2 * 10^5}{4400\sqrt{F_{cu}}} \simeq 10$

٢ - نحسب A_v المساحه التخيليه للقطاع بالكامل $A_v = A_c + (n-1)A_s + (n-1)A_{s'}$

٣ - نحسب $\bar{y}_t = \bar{y}_v$ و تكون من جهه الشد *Tension Side*

٤ - نحسب I_v و هو عزم القصور الذاتى للقطاع التخيلى بالكامل $I_v = I_g$

٥ - نحسب F_{ctr}

$$\boxed{F_{ctr} = 0.6\sqrt{F_{cu}}}$$

٦ - نحسب $M_{cr.}$

$$\boxed{M_{cr.} = \frac{F_{ctr} * I_g}{\bar{y}_t}}$$

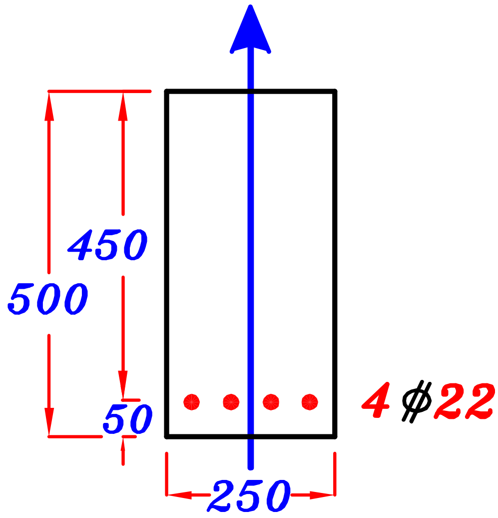
Example.

Data.

Mega Pascal

$$F_{cu} = 25\ N\backslash mm^2 = 25\ Mpa$$

st. 360/520



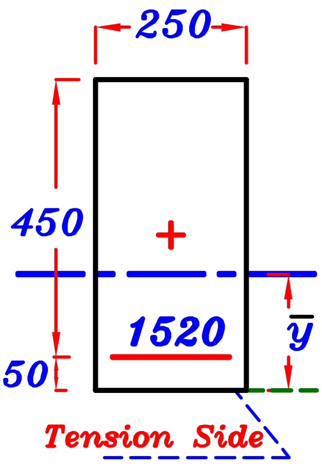
Req.

For the shown Cross-Section

Calculate M_{cr}.

Solution.

$$A_s = 4\phi 22 = 4\left[\frac{\pi * 22^2}{4}\right] = 1520\ mm^2$$



① $$n = \frac{E_s}{E_c} = \frac{2 * 10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1)A_s$$

$$A_v = 250 * 500 + (10-1)(1520) = 138680\ mm^2$$

③ $$\bar{y}_t = \frac{250 * 500 * 250 + (10-1)(1520)(50)}{138680} = 230.27\ mm$$

④ $$I_{gross} = \frac{250 * 500^3}{12} + 250 * 500(250 - 230.27)^2 + (10-1)(1520)(230.27 - 50)^2$$

$$= 3097388472\ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 3097388472}{230.27} = 40353347.9\ N.mm$$

$$= \frac{40353347.9\ N.mm}{10^6} = 40.35\ kN.m$$

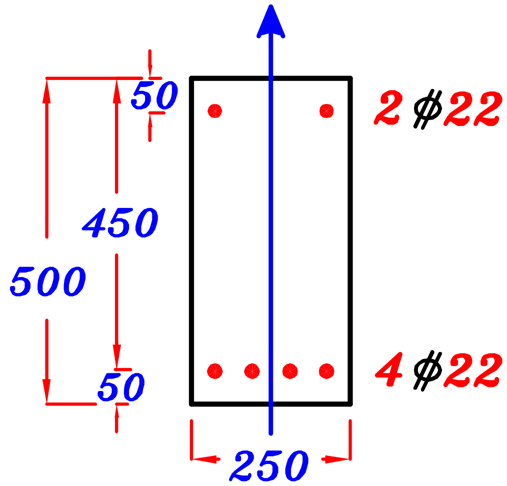
$$\boxed{M_{cr} = 40.35\ kN.m}$$

Example.

Data. $F_{cu} = 25\ N/mm^2 = 25\ Mpa$

st. $360/520$

Req. Calculate M_{cr}.



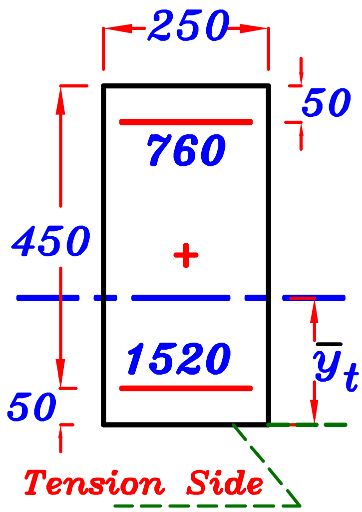
Solution.

$$A_s = 4\phi22 = 4\left[\frac{\pi * 22^2}{4}\right] = 1520\ mm^2$$

$$A_s` = 2\phi22 = 2\left[\frac{\pi * 22^2}{4}\right] = 760\ mm^2$$

IF $A_s` < 0.2\ A_s$ We can neglect $A_s`$

$\because \frac{A_s`}{A_s} = \frac{760}{1520} = 0.50 > 0.2$ $\therefore$ We can't neglect $A_s`$



① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = b*t + (n-1)A_s + (n-1)A_s`$$

$$A_v = 250*500 + (10-1)(1520) + (10-1)(760) = 145520\ mm^2$$

③ $$\bar{y}_t = \frac{250*500*250 + (10-1)(1520)(50) + (10-1)(760)(450)}{145520} = 240.6\ mm$$

④ $$I_{gross} = \frac{250*500^3}{12} + 250*500(250-240.6)^2 + (10-1)(1520)(240.6-50)^2 + (10-1)(760)(450-240.6)^2 = 3412106414\ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

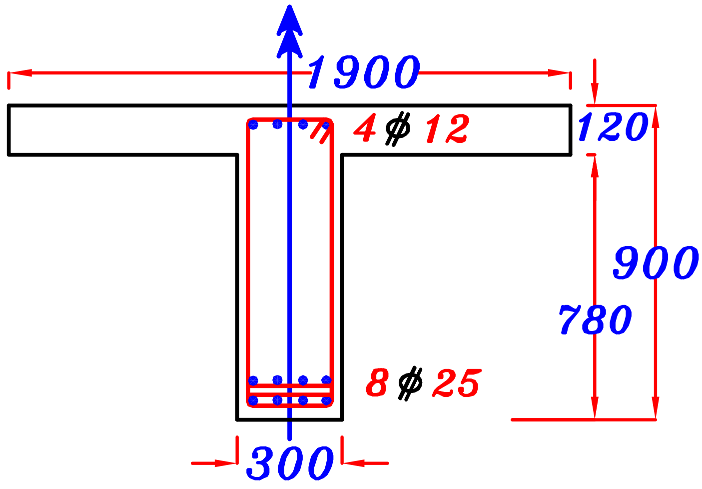
⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 3412106414}{240.6} = 42544967.7\ N.mm$$

$$= \frac{42544967.7\ N.mm}{10^6} = 42.54\ kN.m$$

$$\boxed{M_{cr} = 42.54\ kN.m}$$

Example.

Data.

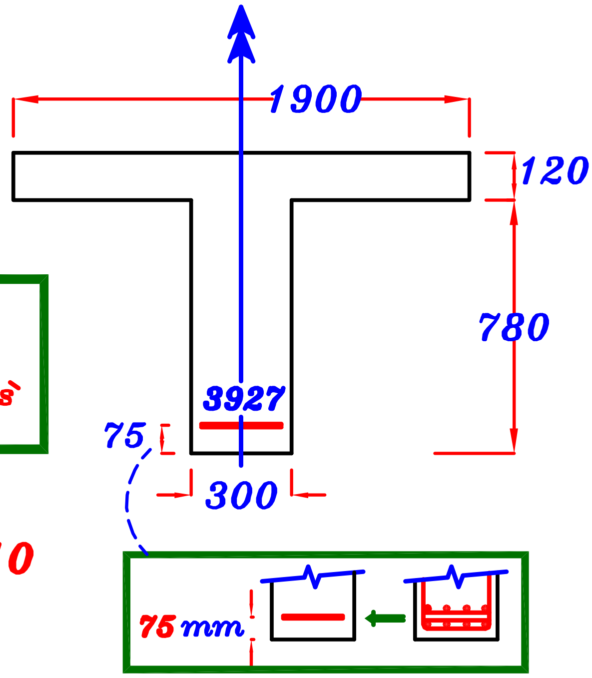


$F_{cu} = 25 \ N/mm^2 = 25 \ Mpa$

st. 360/520

Req. Calculate M_{cr}.

Solution.



$$A_s = 8 \phi 25 = 8 \left[\frac{\pi * 25^2}{4}\right] = 3927 \ mm^2$$

$$A_s` = 4 \phi 12 = 4 \left[\frac{\pi * 12^2}{4}\right] = 452 \ mm^2$$

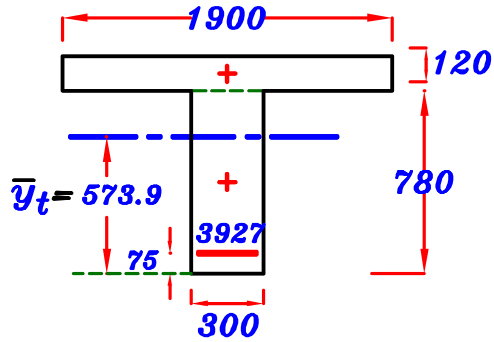
IF $A_s` < 0.2 \ A_s$ We can neglect $A_s`$

$\because \ \frac{A_s`}{A_s} = \frac{531}{3930} = 0.135 < 0.2 \ \therefore$ We can neglect $A_s`$

① $$n = \frac{E_s}{E_c} = \frac{2 * 10^5}{4400 \sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1) A_s = 120 * 1900 + 780 * 300 + (10-1)(3927) = 497343 \ mm^2$$

③ $$\bar{y}_t = \frac{120 * 1900 * (780+60) + 780 * 300 * \left(\frac{780}{2}\right) + (10-1)(3927)(75)}{497343} = 573.9 \ mm$$



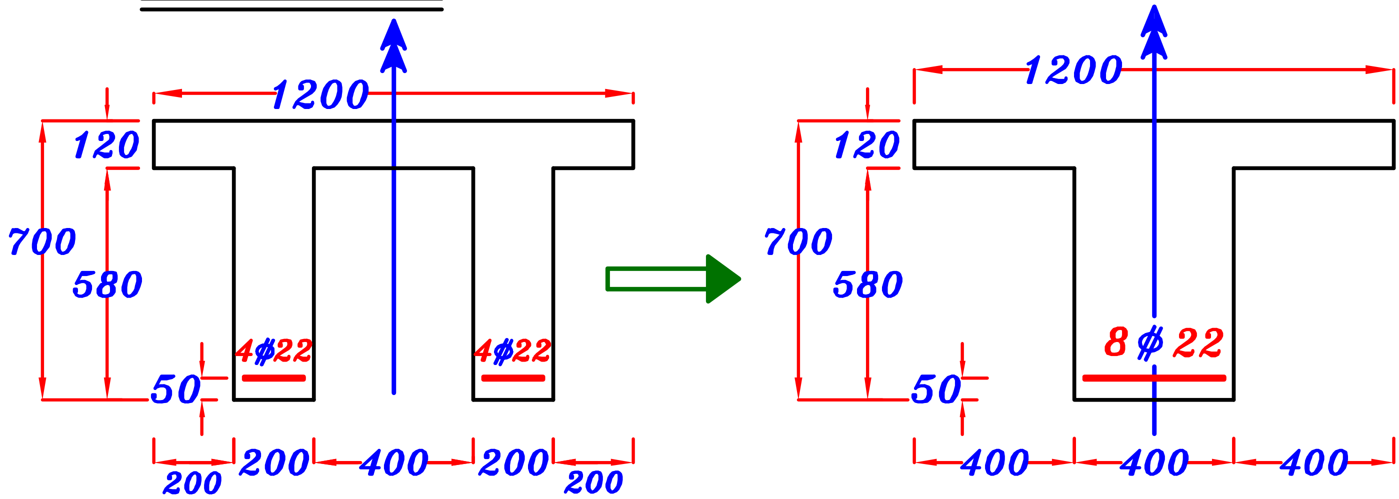
④ $$I_{gross} = \frac{1900 * 120^3}{12} + 1900 * 120 (780+60-573.9)^2 + \frac{300 * 780^3}{12}$$
$$+ 300 * 780 \left(573.9 - \frac{780}{2}\right)^2 + (10-1)(3927)(573.9-75)^2$$
$$= 44992510490 \ mm^4$$

⑤ $$F_{ctr} = 0.6 \sqrt{F_{cu}} = 0.6 \sqrt{25} = 3.0 \ N \backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 44992510490}{573.9} = 235193468.3 \ N.mm$$
$$= 235.19 \ kN.m$$

$$\boxed{M_{cr} = 235.19 \ kN.m}$$

Example.



We can convert the Sec. to an easyer Cross-Sec. and has the same properties. (Area , $\bar{y}$, A_s , c , I & $M_{cr.}$)

Data. $F_{cu} = 25 \ N\backslash mm^2$ st. 360/520

Req. For the shown Cross-Section Calculate $M_{cr.}$

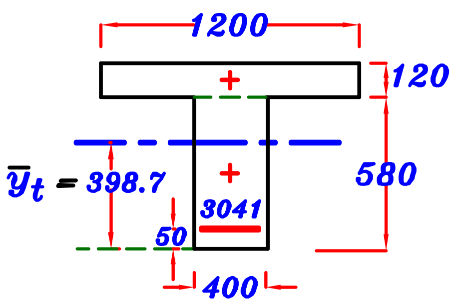
Solution. $A_s = 8\phi 22 = 8\left[\frac{\pi * 22^2}{4}\right] = 3041 \ mm^2$

① $n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$

② $A_v = A_c + (n-1)A_s = 120*1200 + 580*400 + (10-1)(3041) = 403369 \ mm^2$

③ $\bar{y}_t = \frac{1200*120*(580+60) + 580*400*\frac{580}{2} + (10-1)(3041)(50)}{403369}$

$= 398.7 \ mm$



④ $I_{gross} = \frac{1200*120^3}{12} + 1200*120(580+60-398.7)^2 + \frac{400*580^3}{12} + 400*580\left(398.7 - \frac{580}{2}\right)^2$

$+ (10-1)(3041)(398.7-50)^2 = 21130115740 \ mm^4$

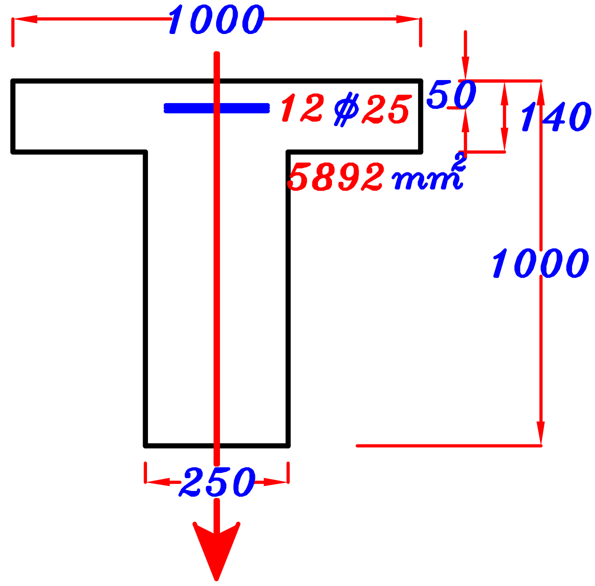
⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0 \ N\backslash mm^2$

⑥ $M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 21130115740}{398.7} = 158992594 \ N.mm$

$= 159.0 \ kN.m.$

$$\boxed{M_{cr} = 159.0 \ kN.m}$$

Example.



Data. $F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

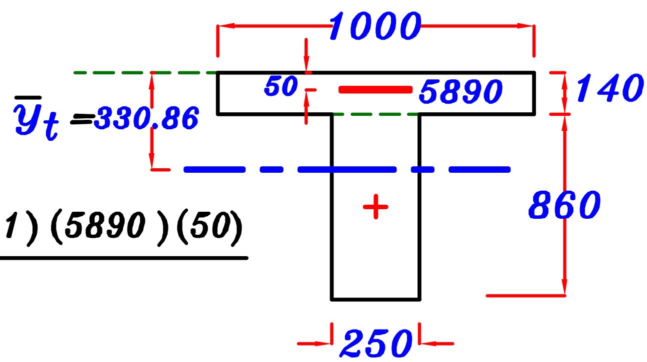
Req. Calculate $M_{cr.}$

Solution.

$$A_s = 12\,\phi 25 = 12\left[\frac{\pi * 25^2}{4}\right] = 5890\ mm^2$$

① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$$

② $$A_v = A_c + (n-1)A_s = 140*1000 + 860*250 + (10-1)(5890) = 408010\ mm^2$$



③ $$\bar{y}_t = \frac{1000*140*70 + 250*860*\left(\frac{860}{2} + 140\right) + (10-1)(5890)(50)}{408010}$$

$$= 330.87\ mm$$

④ $$I_{gross} = \frac{1000*140^3}{12} + 1000*140\,(330.87-70)^2 + \frac{250*860^3}{12} + 250*860\left(\frac{860}{2} + 140 - 330.87\right)^2$$

$$+ (10-1)(5890)(330.87-50)^2 = 39483504630\ mm^4$$

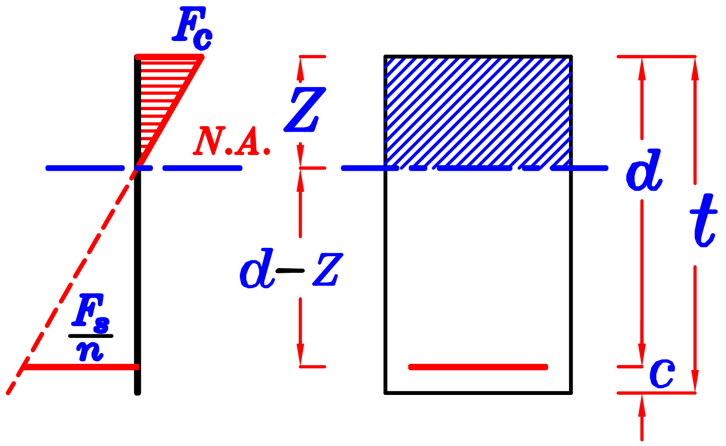
⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.0 * 39483504630}{330.87} = 357997140.5\ N.mm$$

$$= 358.0\ kN.m.$$

$$\boxed{M_{cr} = 358.0\ kN.m}$$

Working Moment (M_w)
OR Allowable Moment



عندما تتشرخ الخرسانه فى منطقه الشد
يتحول القطاع الفعال كما بالشكل
الى خرسانه فى منطقه الضغط
و حديد فى منطقه الشد .

أكبر إجهاد تتحمله الخرسانه فى الضغط F_{cu}

أكبر إجهاد يتحمله الحديد فى الشد أو الضغط F_y

و إذا زادت الإجهادات المؤثره على أى من الخرسانه أو الحديد عن F_{cu} أو F_y يحدث إنهيار للكمره .

لذا فنعمل على أن تكون الإجهادات المؤثره أقل من F_{cu} ، F_y حتى لا يحدث إنهيار للكمره .

و هذه الإجهادات تسمى (الإجهادات المسموح بها) **Allowable Stresses**

أى أنها أكبر إجهادات نسمح بها لكى تؤثر على الحديد و الخرسانه مع ضمان عدم الإنهيار.

Allowable Stresses For Concrete $= F_c$

Allowable Stresses For Steel $= F_s$

F_{cu} $(N\backslash mm^2)$	18.0	20.0	25.0	30.0	35.0	40.0
F_c $(N\backslash mm^2)$	7.0	8.0	9.5	10.5	11.5	12.5

F_y $(N\backslash mm^2)$	240	360	400
F_s $(N\backslash mm^2)$	140	200	220

Egyptian Code Page (5−2)

أنواع الإجهادات	المصطلحات	إجهادات التشغيل وفقاً لرتب الخرسانة حسب مقاومتها المميزة للمكعب القياسي بعد ٢٨ يوماً (ن/مم٢)			
مقاومة الخرسانة المميزة (الرتبة)	f_{cu}	18	20	25	30
الضغط المحوري ($e=e_{min}$)	f_{co}	4.5	5	6	7
الانحناء أو الضغط كبير اللامركزية	f_c	7.0	8.0	9.5	10.5
القص					
مقاومة الخرسانة للقص					
بدون تسليح في البلاطات والقواعد	q_c	0.7	0.8	0.9	0.9
بدون تسليح في الأعضاء الأخرى	q_c	0.5	0.6	0.7	0.7
وجود تسليح جذعي في جميع الأعضاء (القص واللي معا)	q_2	1.5	1.7	1.9	2.1
القص الثاقب	q_{cp}	0.7	0.8	0.9	1.0
الصلب الفولاذ					
1-صلب طري 240/350	f_s	140	140	140	140
2- صلب 280/450		160	160	160	160
3-صلب 360/520		200	200	200	200
4-صلب 400/600		220	220	220	220
5-الشبك الملحوم 450/520 أملس		160	160	160	160
ذو النتوءات أو ذو العضات		220	220	220	220

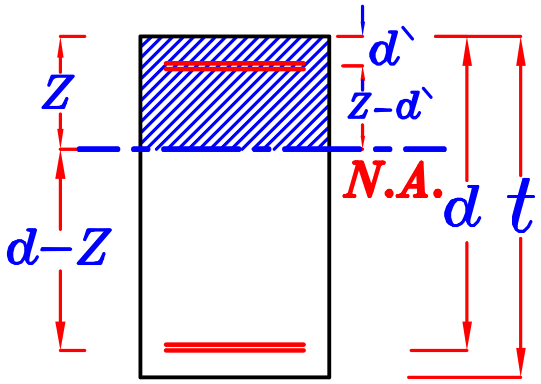
و تعريف ال Working Moment (M_w)

هو العزم الذى يوصل الاجهادات على أى من الحديد أو الخرسانه الى *Allowable Stresses*

فى المسأله عندما يعطينا القطاع و يطلب تحديد M_w

تكون خطوات الحل كالأتى :ـ

١ ـ نأخذ $n \simeq 15$ *Modular ratio after cracking*



٢ ـ لتحديد مكان ال *N.A.*

نحسب قيمه Z و تكون من جهه الضغط

و ذلك بأن نأخذ $S_{nv} = Zero$

٣ ـ نحسب قيمه I_{nv} و هو عزم القصور الذاتى للقطاع الشغال حول ال *N.A.*

٤ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الخرسانه فى الضغط F_c

$$\boxed{M_{wc} = \frac{F_c * I_{nv}}{Z}}$$

٥ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الحديد فى الشد F_s

$$\boxed{M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}}$$

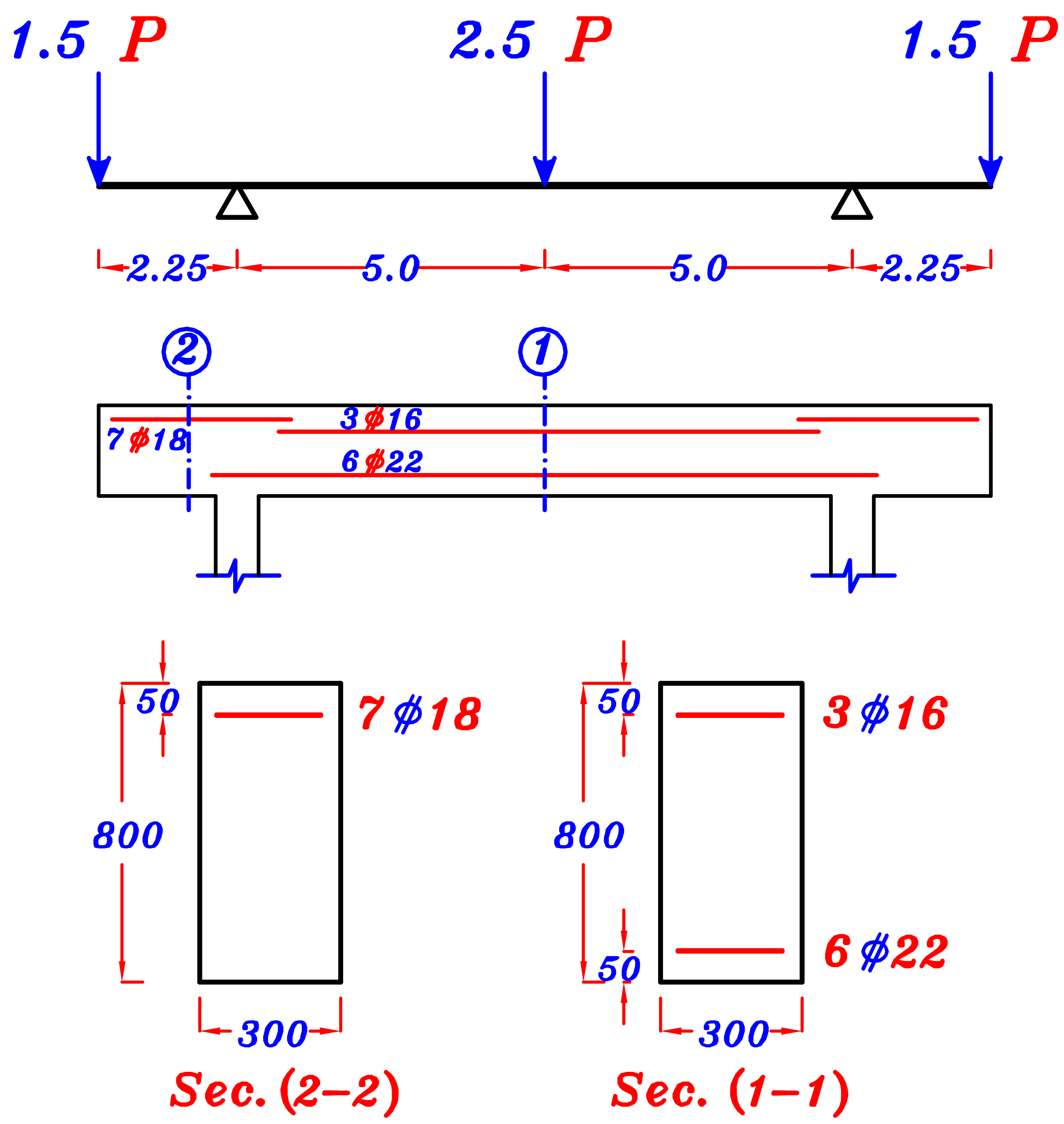
٦ ـ نحسب قيمه العزم الذى يجعل الإجهادات على الحديد فى الضغط $F_{s`}$

(ممكن إهمال هذه الخطوه)

$$\boxed{M_{ws`} = \frac{\left(\frac{F_{s`}}{n}\right) * I_{nv}}{Z-d`}}$$

٧ ـ نأخذ القيمه الاقل من M_{wc} , M_{ws} , $M_{ws`}$ فتكون هى M_w للقطاع

Example.



Data.

neglecting O.W.

$F_{cu} = 25 \ N\backslash mm^2$ $F_y = 360 \ N\backslash mm^2$

Req.

Fined the allowable working loads (P_w) acting on the beam.

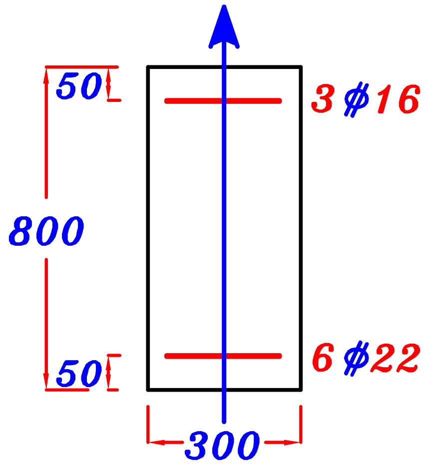
Allowable stresses

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$

Solution.

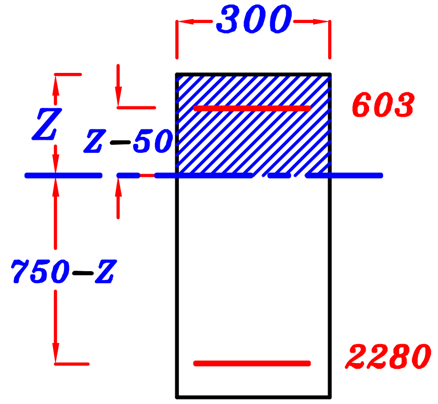
Sec. ①



$$A_s = 6\,\phi 22 = 8\left[\frac{\pi * 22^2}{4}\right] = 2280\ mm^2$$

$$A_s` = 3\,\phi 16 = 3\left[\frac{\pi * 16^2}{4}\right] = 603\ mm^2$$

$$\boxed{\because\ \frac{A_s`}{A_s} = \frac{603}{2280} = 0.26 > 0.2 \quad \therefore\ \text{We can't neglect } A_s`}$$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_s`\,(Z - d`) = n\,A_s\,(d - Z)$$

$$300\,(Z)\left(\frac{Z}{2}\right) + (14)(603)\,(Z - 50) = (15)\,(2280)\,(750 - Z)$$

$$\boxed{Z = 298.3\ mm}$$

③ Get $I_{nv} = \frac{b Z^3}{3} + (n-1)\,A_s`\,(Z - d`)^2 + n\,A_s\,(d - Z)^2$

$$I_{nv} = \frac{300\,(298.3)^3}{3} + (14)\,(603)\,(298.3 - 50)^2 + (15)\,(2280)\,(750 - 298.3)^2$$

$$= 10152758140\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 10152758140}{298.3} = 323336246.6\ N.mm$

$$= 323.33\ kN.mm$$

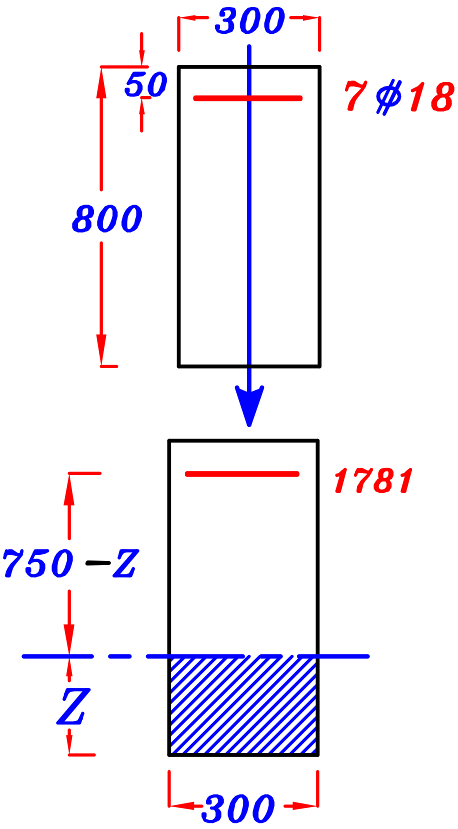
⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{200}{15}\right) * 10152758140}{750 - 298.3} = 299690300\ N.mm$

$$= 299.7\ kN.m$$

⑥ $\boxed{M_{w1} = 299.7\ kN.m}$

Sec. ②

$A_s = 7\phi 18 = 7\left[\frac{\pi * 18^2}{4}\right] = 1781\ mm^2$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$b(Z)\left(\frac{Z}{2}\right) = n\,A_s(d-Z)$$

$$300(Z)\left(\frac{Z}{2}\right) = (15)(1781)(750-Z)$$

$$\boxed{Z = 287.1\ mm}$$

③ Get $I_{nv} = \frac{bZ^3}{3} + n\,A_s(d-Z)^2$

$$I_{nv} = \frac{300\ (287.1)^3}{3} + (15)(1781)(750-287.1)^2 = 8090856524\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 8090856524}{287.1} = 267722525\ N.mm$

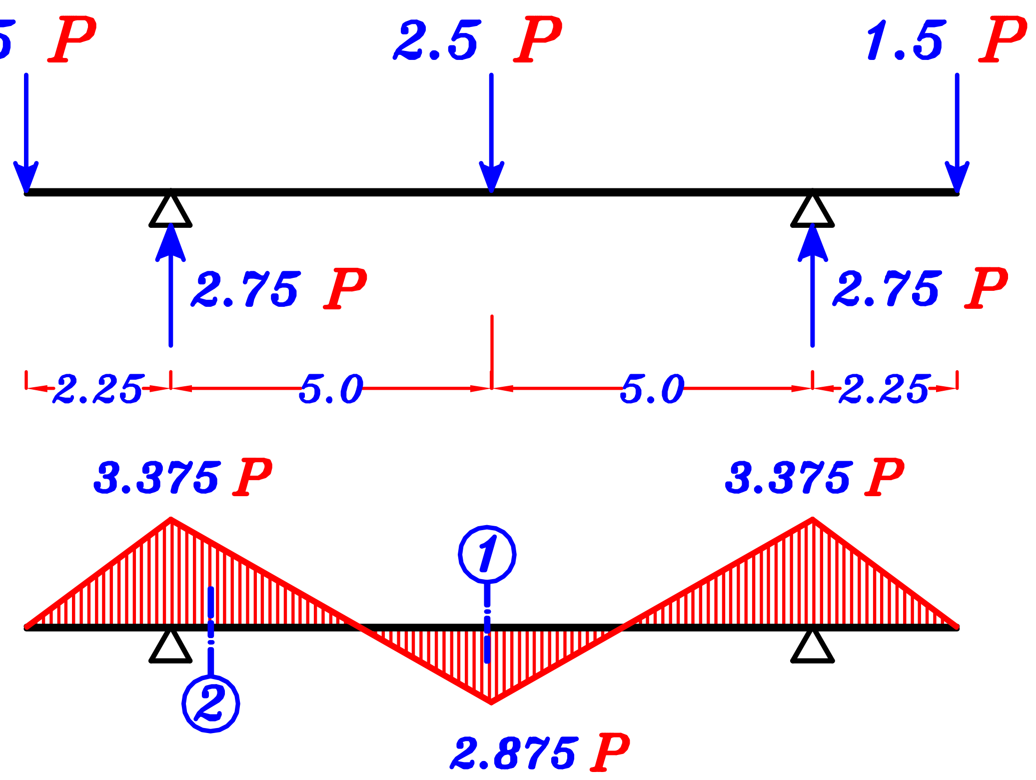
$= 267.72\ kN.m$

⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 8090856524}{750-287.1} = 233048362\ N.mm$

$= 233.05\ kN.m$

⑥ $\boxed{M_{w2} = 233.05\ kN.m}$

Actual Moment.



Sec. 1 $M_{act.} = 2.875\ P$

To Get $P_w \longrightarrow M_{act.} = M_w$

$\therefore\ 2.875\ P_w = 299.7\ kN.m \longrightarrow P_{w1} = 104.24\ kN$

Sec. 2 $M_{act.} = 3.375\ P$

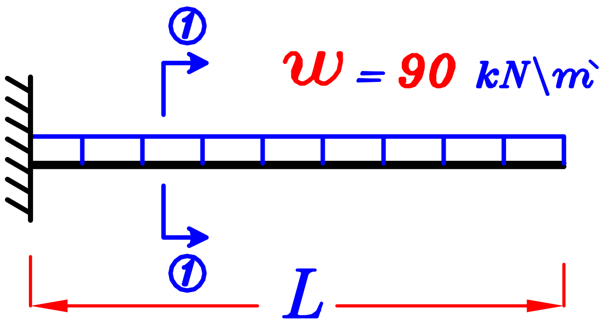
To Get $P_w \longrightarrow M_{act.} = M_w$

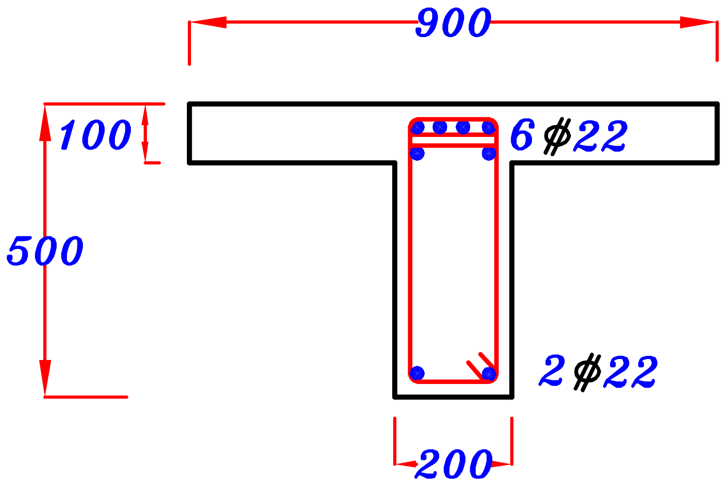
$\therefore\ 3.375\ P_w = 233.05\ kN.m \longrightarrow P_{w2} = 69.05\ kN$

P_w For all the beam is the least one of P_{w1}, P_{w2}

$$\boxed{P_w = 69.05\ kN}$$

Example.





Sec. (1–1)

Data.

$F_{cu} = 25 \; N\backslash mm^2 \qquad F_y = 360 \; N\backslash mm^2$

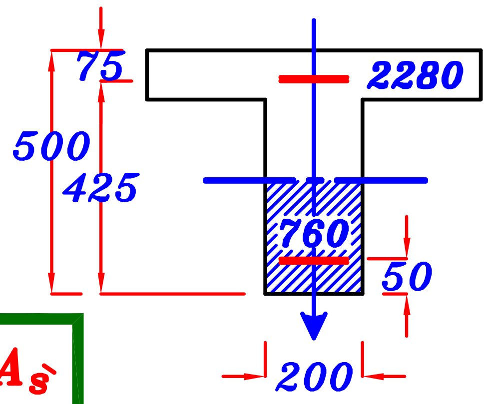
Req.

Fined the maximum design length For the cantilever.

Solution.

$$A_s = 6\,\phi 22 = 6\left[\frac{\pi * 22^2}{4}\right] = 2280 \; mm^2$$

$$A_s` = 2\,\phi 22 = 2\left[\frac{\pi * 22^2}{4}\right] = 760 \; mm^2$$



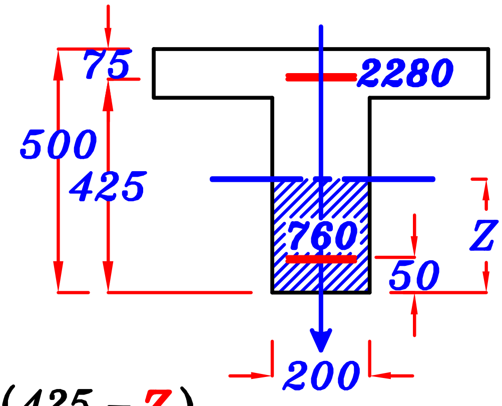
$\because \frac{A_s`}{A_s} = \frac{760}{2280} = 0.33 > 0.2 \quad \therefore$ We can't neglect $A_s`$

Allowable stresses

$$F_{cu} = 25 \; N\backslash mm^2 \longrightarrow F_c = 9.5 \; N\backslash mm^2$$

$$F_y = 360 \; N\backslash mm^2 \longrightarrow F_s = 200 \; N\backslash mm^2$$

① Take $n = 15$



② Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_{s`}\,(Z - d`) = n\,A_s\,(d - Z)$$

$$200\,(Z)\left(\frac{Z}{2}\right) + (14)(760)(Z - 50) = (15)(2280)(425 - Z)$$

$$\boxed{Z = 224.0 \text{ mm}}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)\,A_{s`}\,(Z - d`)^2 + n\,A_s\,(d - Z)^2$

$$I_{nv} = \frac{200\,(224.0)^3}{3} + (14)(760)(224.0 - 50)^2 + (15)(2280)(425 - 224.0)^2$$

$$= 2453145773 \text{ mm}^4$$

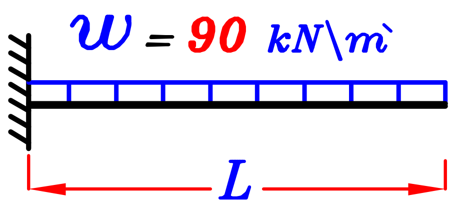
④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 2453145773}{224.0} = 104039664.5 \text{ N.mm}$

$= 104.04 \text{ kN.m}$

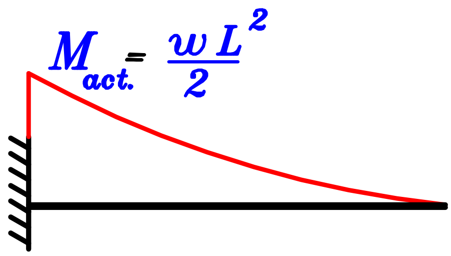
⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 2453145773}{425 - 224.0} = 162729404.5 \text{ N.mm}$

$= 162.73 \text{ kN.m}$

⑥ $\boxed{M_w = 104.04 \text{ kN.m}}$



Actual Moment =



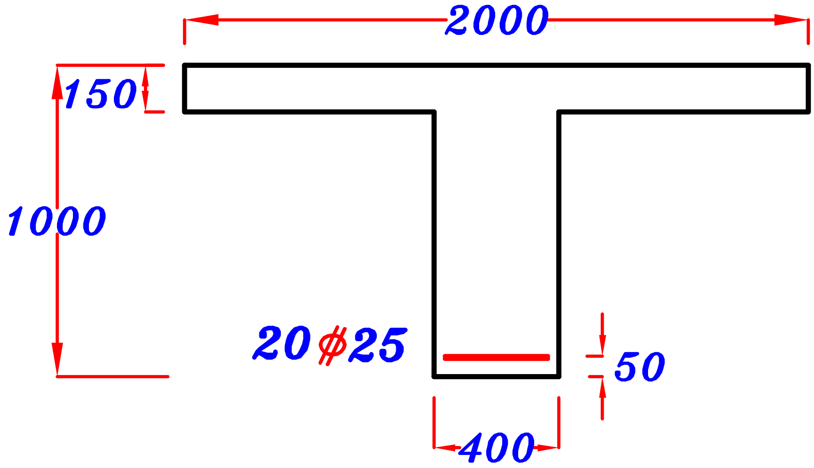
$$M_{act.} = \frac{wL^2}{2} = \frac{90\,L^2}{2} = 45\,L^2$$

To get the maximum design length $= L_w$

$$M_{act.} = M_w$$

$$45\,L^2 = 104.04 \longrightarrow \boxed{L = 1.52 \text{ m}}$$

Example.



Data.

$F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$

Req. Calculate M_w

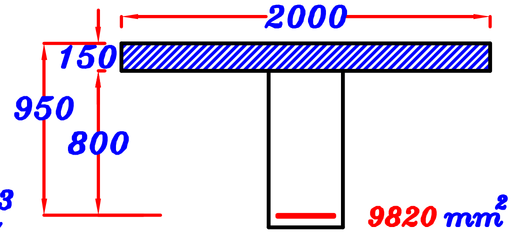
$$A_s = 20\phi 25 = 20\left[\frac{\pi * 25^2}{4}\right] = 9817\ mm^2$$

Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

To know IF Z is bigger or smaller than the Flange thickness $= 150\ mm$

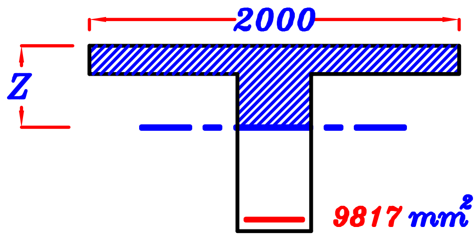


$S_{nv.}(above) = 150 * 2000 * (75) = 22500000\ mm^3$

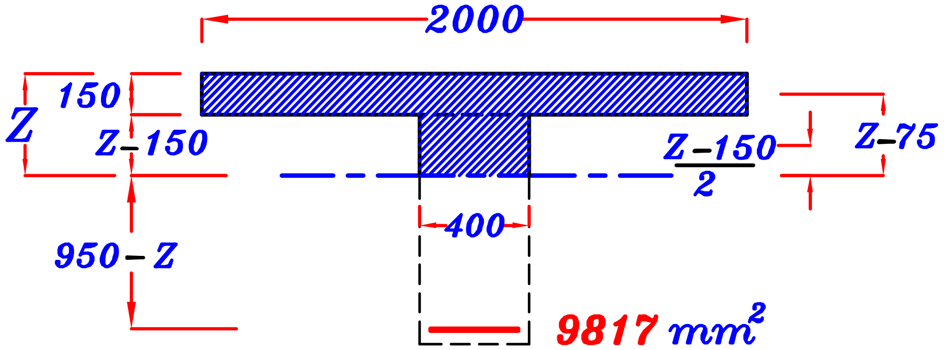
$S_{nv.}(under) = 15 * 9817 * (800) = 117804000\ mm^3$

$\because\ S_{nv.}(under) > S_{nv.}(above)$

$\therefore\ Z > 150\ mm$



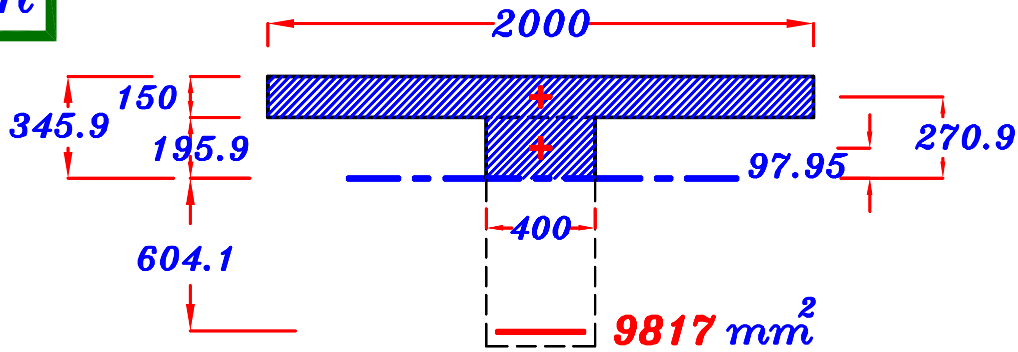
① Take $n = 15$



② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$(2000)(150)(Z-75) + (400)(Z-150)\left(\frac{Z-150}{2}\right) = (15)(9817)(950-Z)$$

$$\boxed{Z = 345.9 \ mm}$$



③ $$I_{nv} = \frac{2000(150)^3}{12} + (2000)(150)(270.9)^2 + \frac{400(195.9)^3}{3} + (15)(9817)(604.1)^2 = 77319715230 \ mm^4$$

④ $$M_{wc} = \frac{\left(\frac{2}{3}\right) F_c * I_{nv}}{Z} \quad \text{-------} \quad T\text{-}Sec.$$

$$= \frac{\left(\frac{2}{3}\right) 9.5 * 77336137390}{345.9} = 1416003287 \ N.mm$$

$$= 1416.0 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}$$

$$= \frac{\left(\frac{200}{15}\right) * 77336137390}{950 - 345.9} = 1706916899 \ N.mm$$

$$= 1706.91 \ kN.m$$

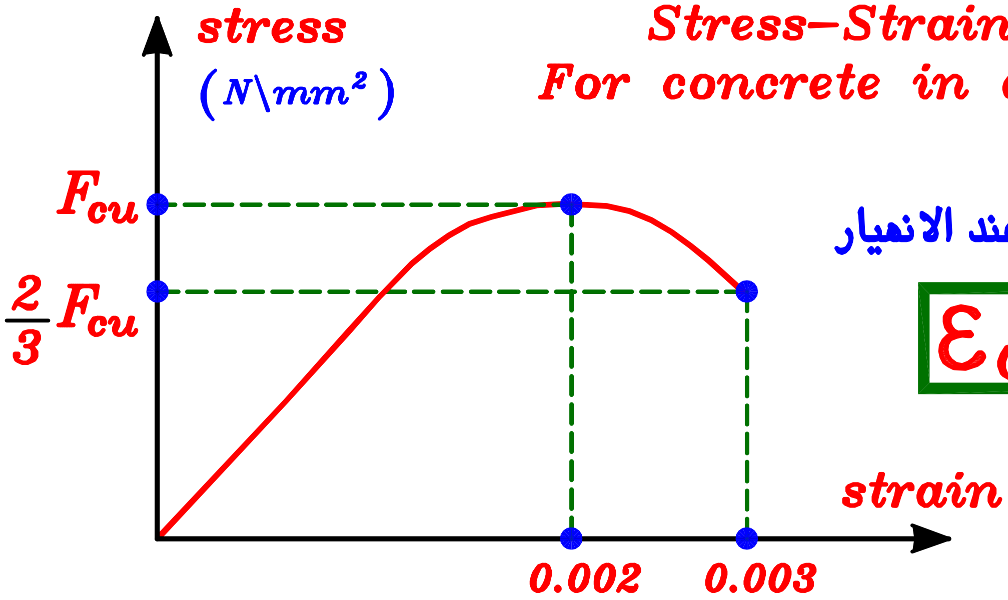
⑥ $$\boxed{M_w = 1416.0 \ kN.m}$$

(M_{ult})

Introduction of Ultimate Moment.

Types of Failure For Sections subjected to B.M. only.

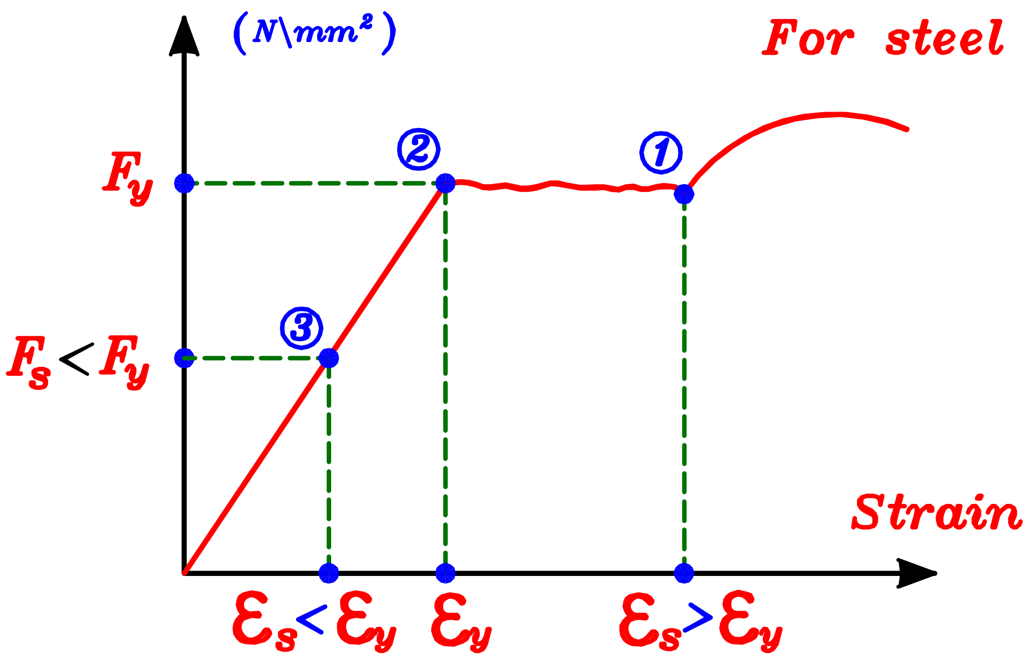
Stress–Strain curve For concrete in compression



استطاله الخرسانه عند الانهيار

$$\boxed{\varepsilon_c \simeq 0.003}$$

Stress–Strain curve For steel in tension



$$\varepsilon_s = \frac{F_s}{E_s} = \frac{F_s}{2*10^5} \quad , \quad \varepsilon_y = \frac{F_y}{E_s} = \frac{F_y}{2*10^5}$$

when $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$

Types of Sections at Failure.

① Under Reinforced Sections. كميه الحديد قليله

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y
بينما لم يصل الاجهاد علىالخرسانه الى أقصى مقاومه لها F_{cu} .

Have a (Ductile Failure)
or (Tension Failure)

إنهيار غير مفاجئ

② Balanced Sections. كميه الحديد متوسطه

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y في نفس الوقت
الذى يصل فيه الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu} .

Have a (Brittle Failure)
or (Balanced Failure)

انهيار مفاجئ

③ Over Reinforced Sections. كميه الحديد كبيره

و فيه يصل الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu}
قبل أن يصل الاجهاد على الحديد الى أقصى مقاومه له F_y .

Have a (Brittle Failure)
or (Balanced Failure)

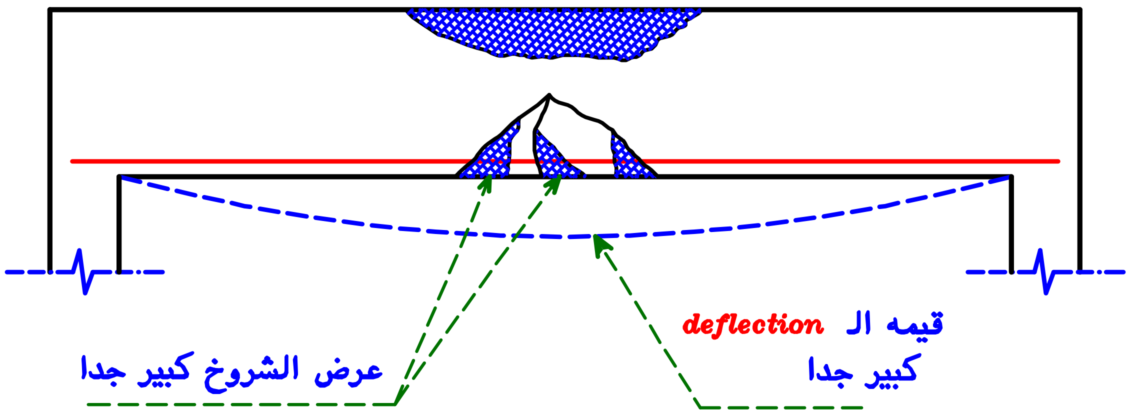
انهيار مفاجئ

① *Under Reinforced Sections.*

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y
بينما لم يصل الاجهاد علىالخرسانه الى أقصى مقاومه لها F_{cu} .

أى يزيد عرض الشروخ كثيرا قبل حدوث الإنهيار
(أى قبل أن تتكسر الخرسانه من جهه الضغط)
و هذا الإنهيار هو المفضل لأنه **إنهيار غير مفاجئ** .

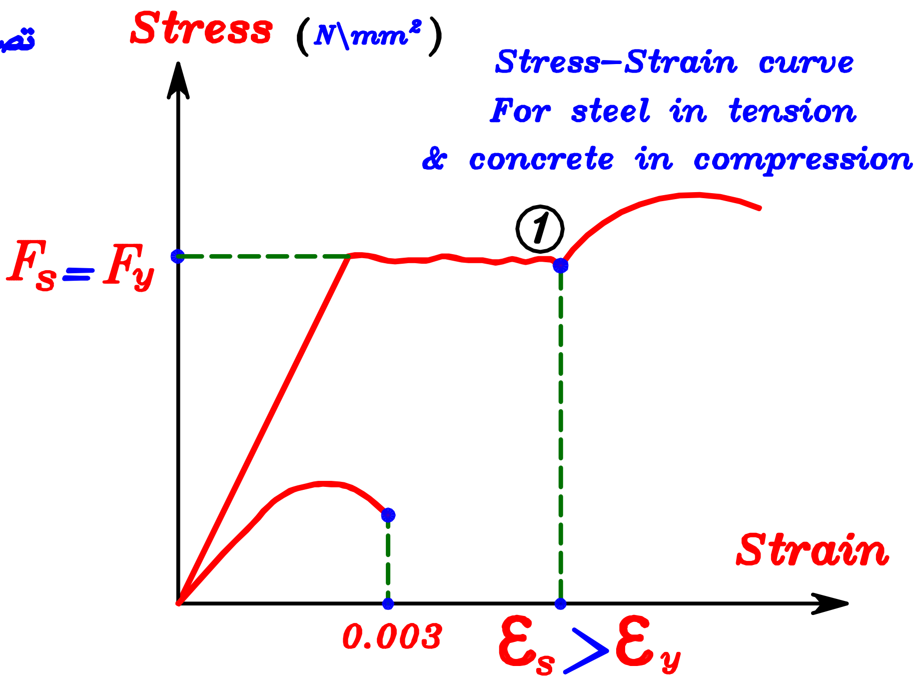
(***Ductile Failure***)



و يسمى *Under Reinforced Section.* لأن كميه الحديد به تكون قليله نسبياً .

F_{cu} تصل الخرسانه الى أقصى إجهاد لها
F_y بعد وصول الحديد إلى

$$\boxed{\begin{gathered}\varepsilon_s > \varepsilon_y \\ F_s = F_y \\ \varepsilon_c = 0.003\end{gathered}}$$



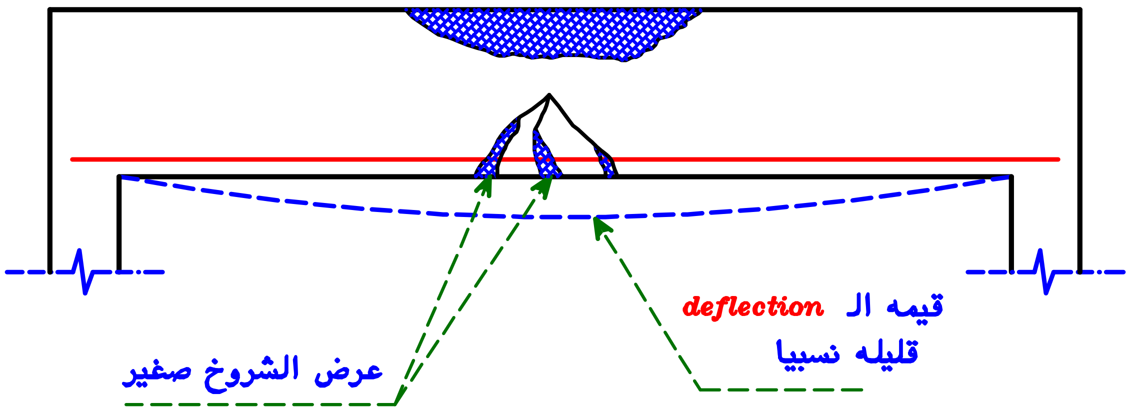
② *Balanced Section.*

و فيه يصل الاجهاد على الحديد الى أقصى مقاومة له F_y فى نفس الوقت الذى يصل فيه الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu} .

و يحدث الإنهيار بإنكسار الخرسانه من جهه الضغط .

و هذا الإنهيار غير مفضل لأنه **إنهيار مفاجئ** .

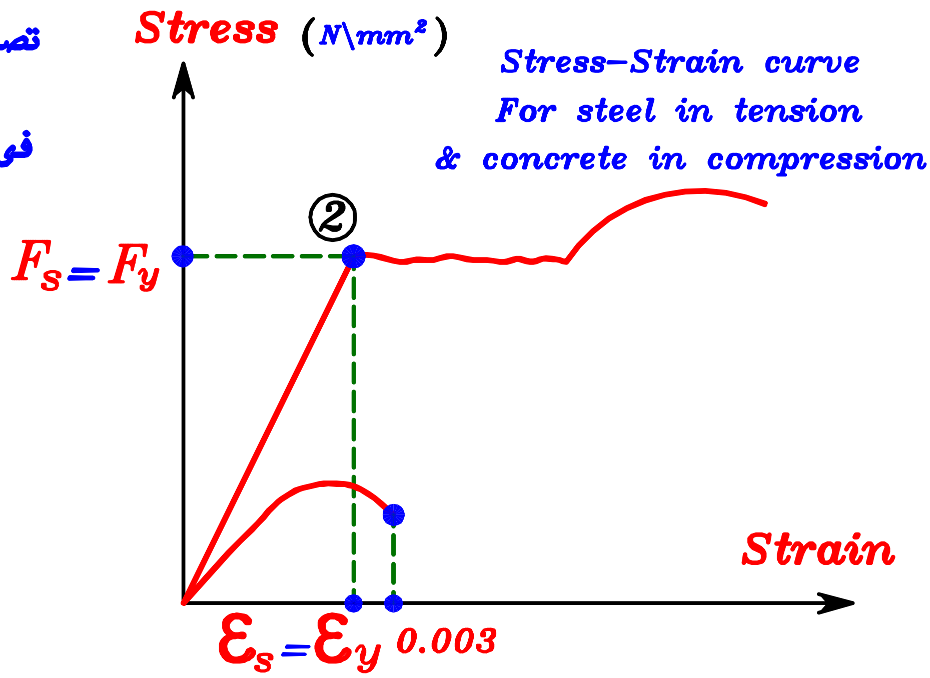
(*Brittle Failure*)



و يسمى *Balanced Section.* لأنّ الخرسانه و الحديد يصلوا الى مرحله الانهيار فى نفس الوقت تماماً (و هذه حاله نادره الحدوث فى الواقع)

F_{cu} تصل الخرسانه الى أقصى إجهاد لها فى نفس وقت وصول الحديد إلى F_y

$$\boxed{\begin{aligned} \varepsilon_s &= \varepsilon_y \\ F_s &= F_y \\ \varepsilon_c &= 0.003 \end{aligned}}$$



③ Over Reinforced Sections.

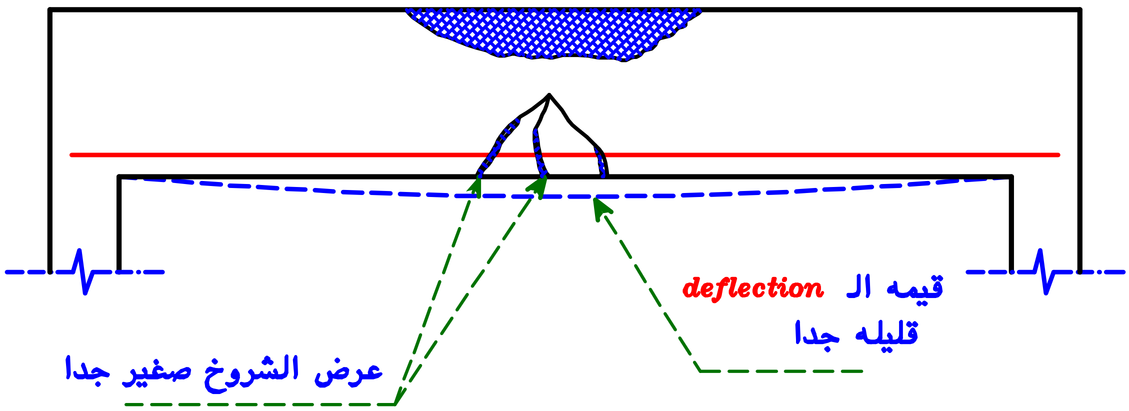
و فيه يصل الاجهاد على الخرسانه الى أقصى مقاومه لها F_{cu}

قبل أن يصل الاجهاد على الحديد الى أقصى مقاومه له F_y .

و يكون عرض الشروخ صغير جداً قبل إنهيار الخرسانه فى الضغط .

و هذا النوع من الإنهيار سيئ جدا لأنه لا يعطى أى مؤشر قبل الإنهيار .

(Brittle Failure)

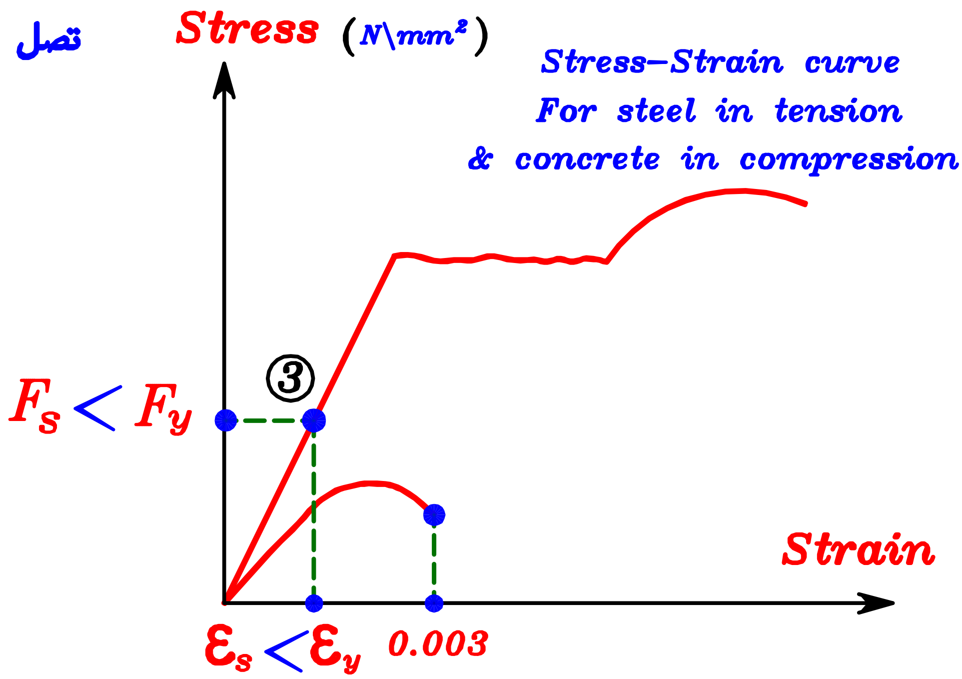


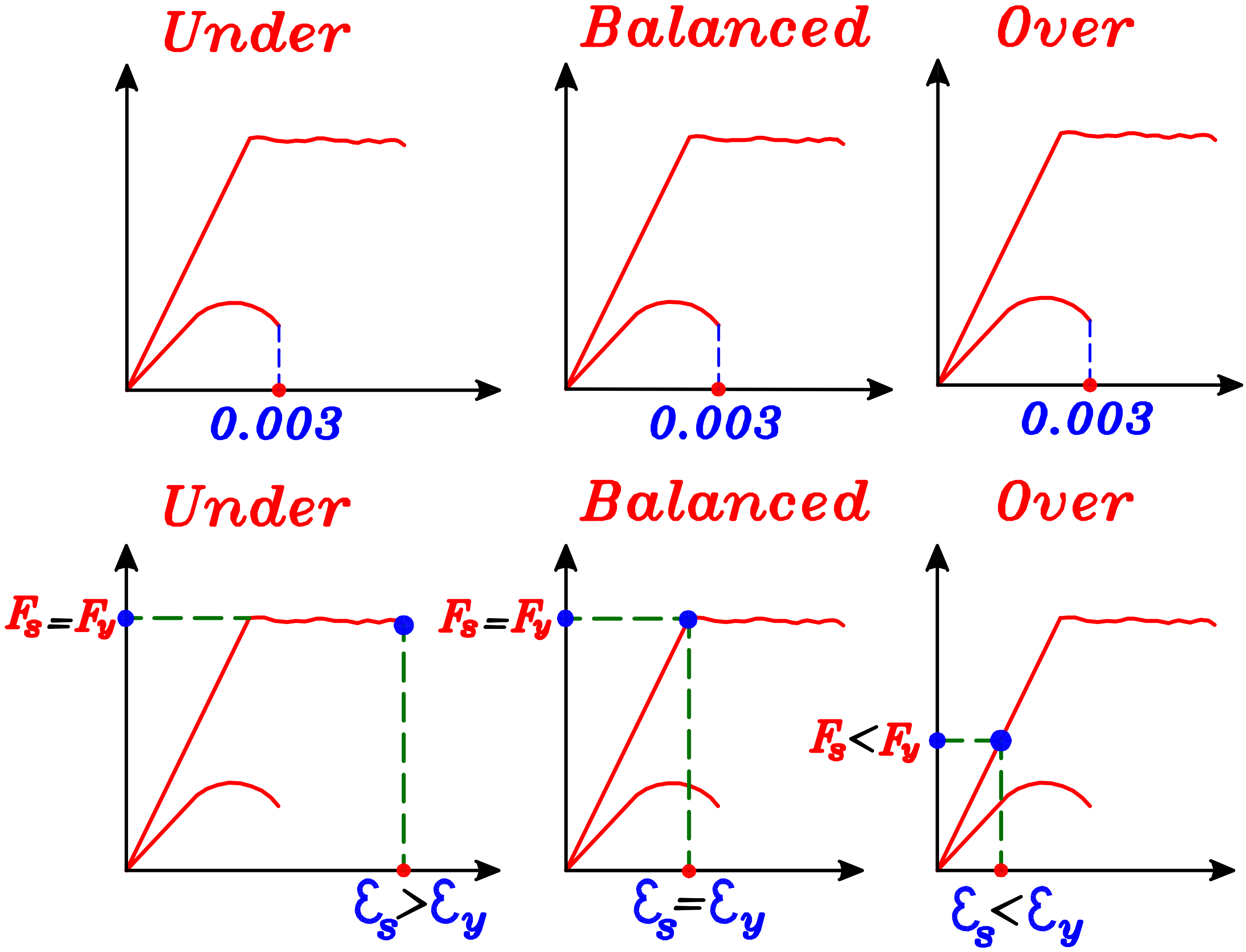
و يسمى *Over Reinforced Section.* لأن كميه الحديد به تكون كبيره .

F_{cu} تصل الخرسانه الى أقصى إجهاد لها

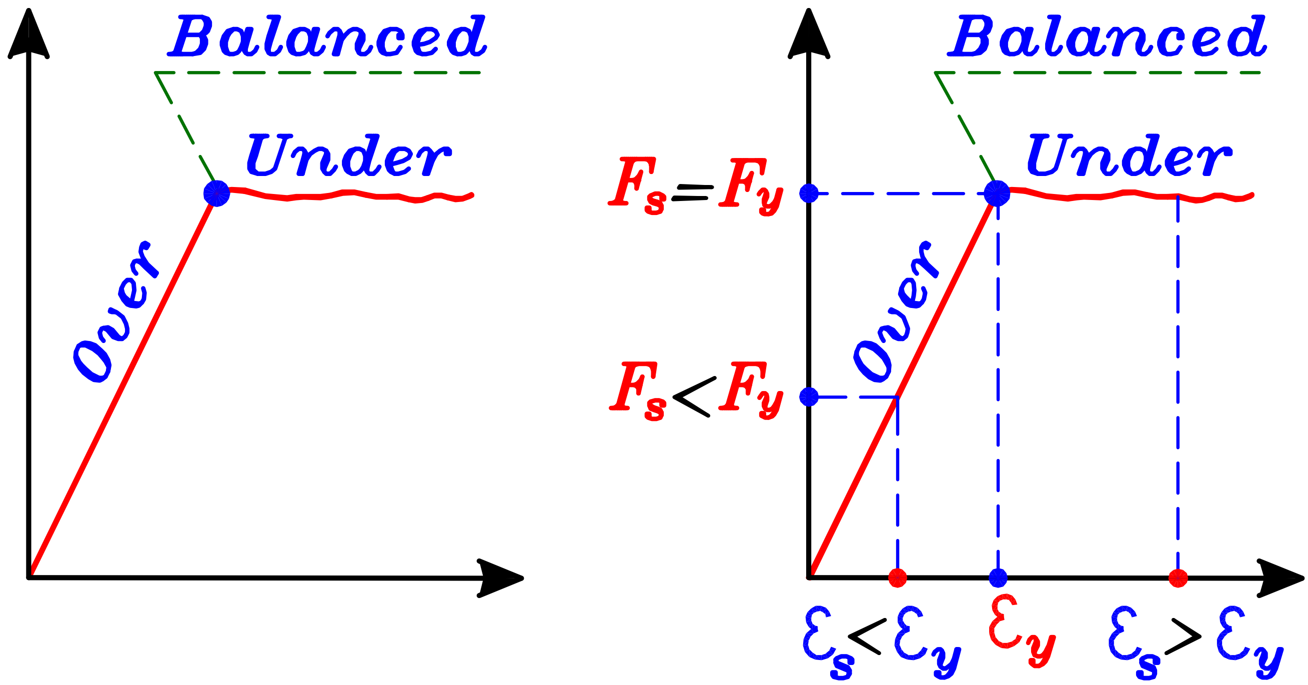
F_y قبل وصول الحديد إلى

$$\boxed{\begin{array}{l} \varepsilon_s < \varepsilon_y \\ F_s < F_y \\ \varepsilon_c = 0.003 \end{array}}$$




Under
Balanced
Over
0.003
0.003
0.003
Under
Balanced
Over
$F_s=F_y$
$F_s=F_y$
$F_s<F_y$
$\varepsilon_s>\varepsilon_y$
$\varepsilon_s=\varepsilon_y$
$\varepsilon_s<\varepsilon_y$

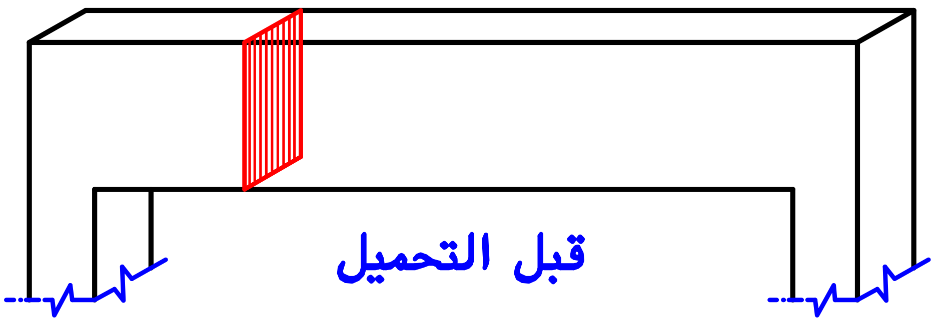
حفظ

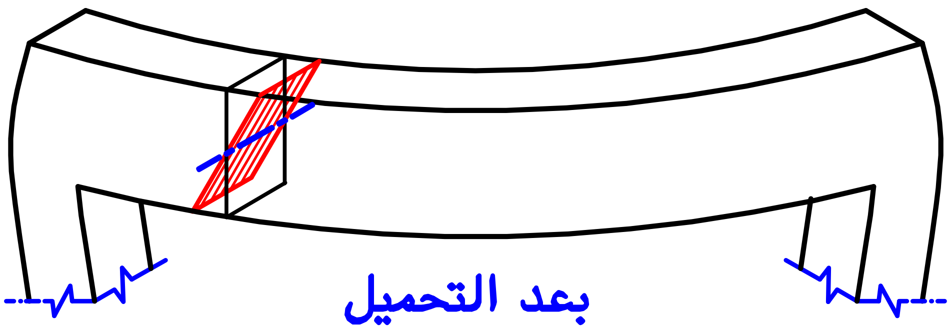

Balanced
Under
Over
Balanced
Under
Over
$F_s=F_y$
$F_s<F_y$
$\varepsilon_s<\varepsilon_y$
ε_y
$\varepsilon_s>\varepsilon_y$

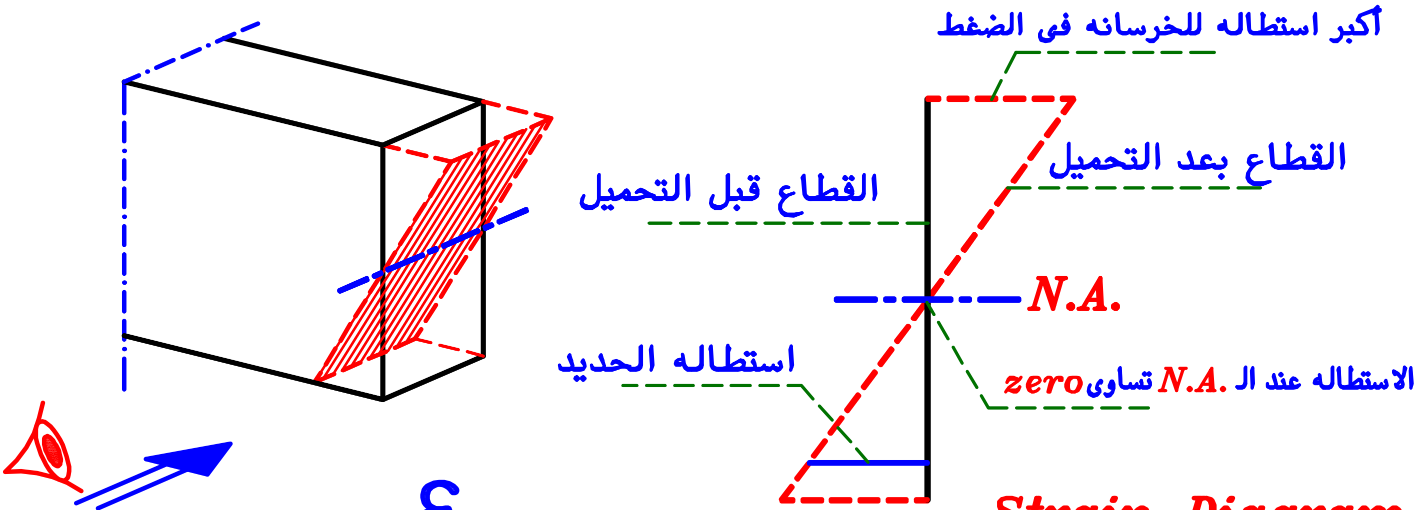
Strain Diagram.

Elastic Theory

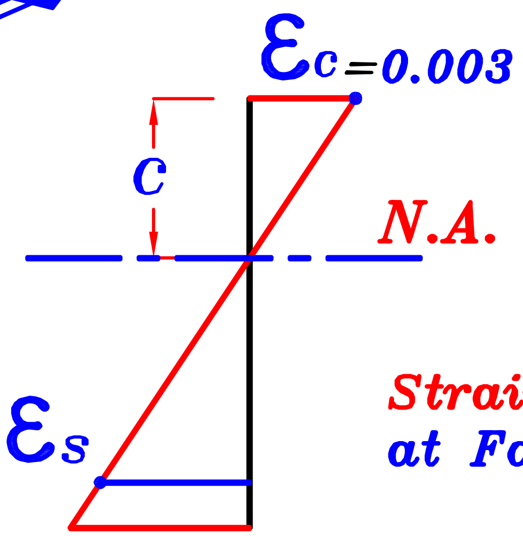
هى نظريه تعتمد على أن شكل القطاع المستوى قبل التحميل يظل مستوى بعد التحميل .





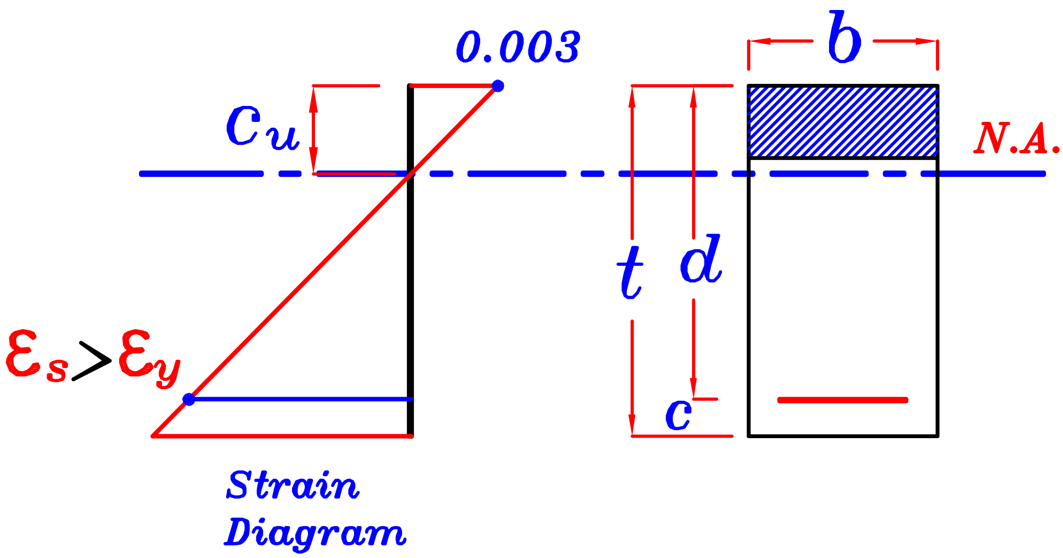


Strain Diagram.

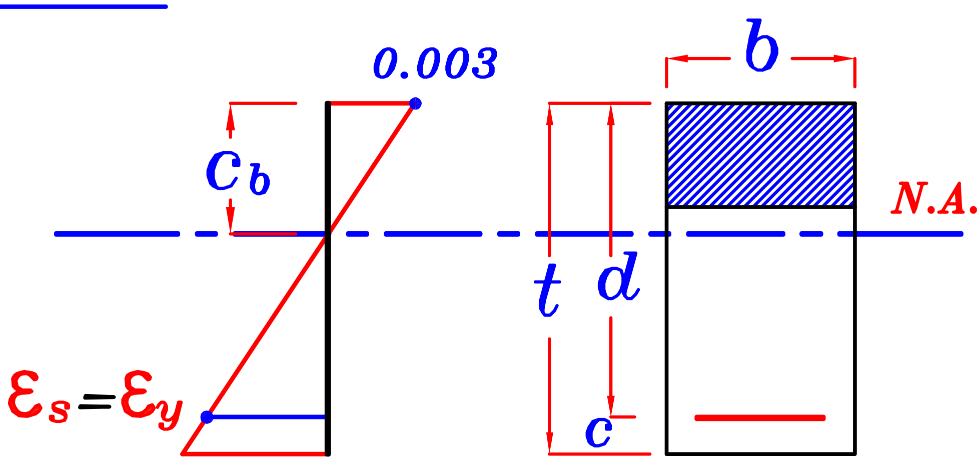


Strain Diagram.
at Faliure

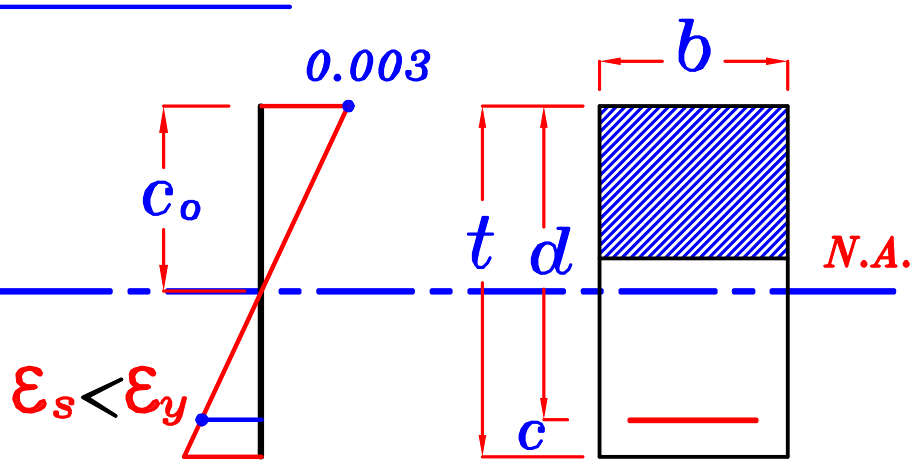
① Under Reinforced Sections.



② Balanced Sections.



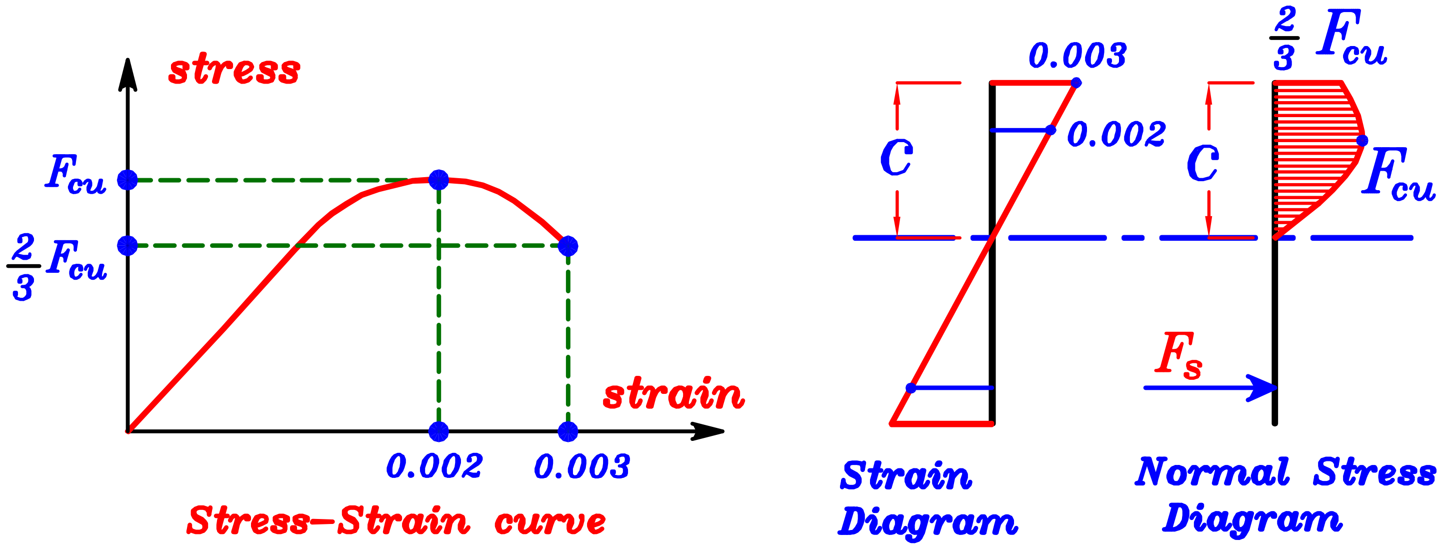
③ Over Reinforced Sections.



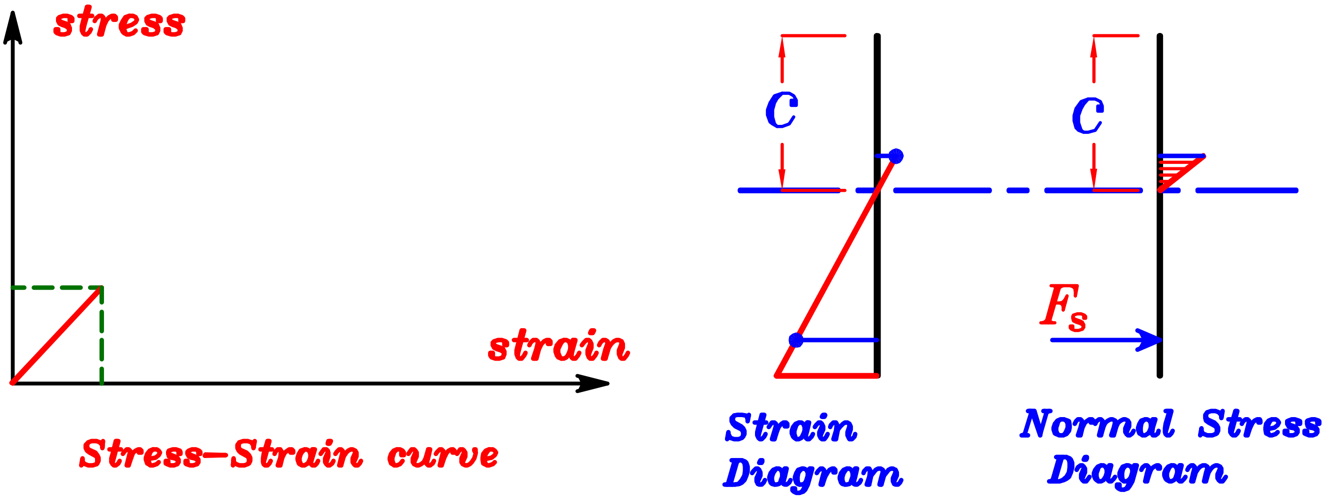
note $\boxed{c_u < c_b < c_o}$

Stress Diagram.

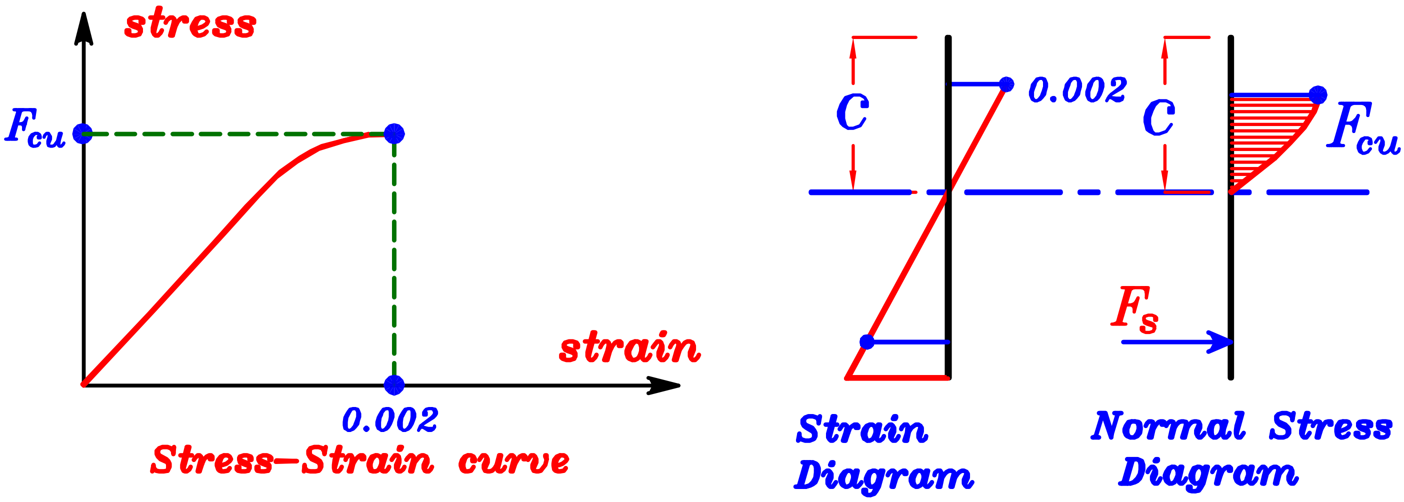
ممكن استنتاج شكل ال *Normal Stress diagram*
من شكل كلا من *Strian diagram* و ال *Stress–Strain curve*



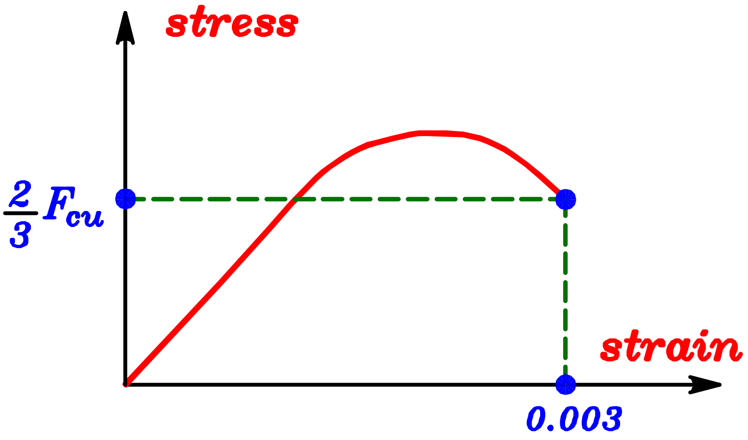
فى البدايه عندما كان ال **Strain** قليل كان ال **stress** قليل و كان فى البدايه خط مستقيم



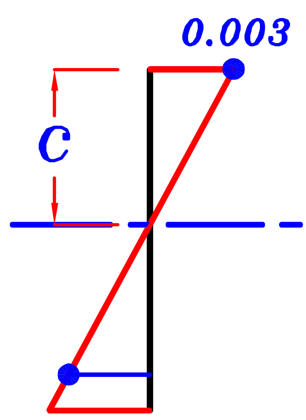
عند وصول ال **Strain** الى قيمه 0.002 يكون ال **Stress** أخذ شكل منحنى و وصل الى قيمه F_{cu}



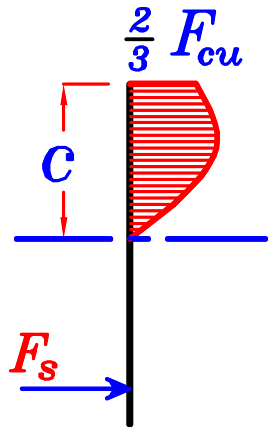
عند وصول ال Strain الى قيمه 0.003 تبدأ الخرسانه فى الانهيار و يكون ال Stress و صل الى قيمه $\frac{2}{3} F_{cu}$



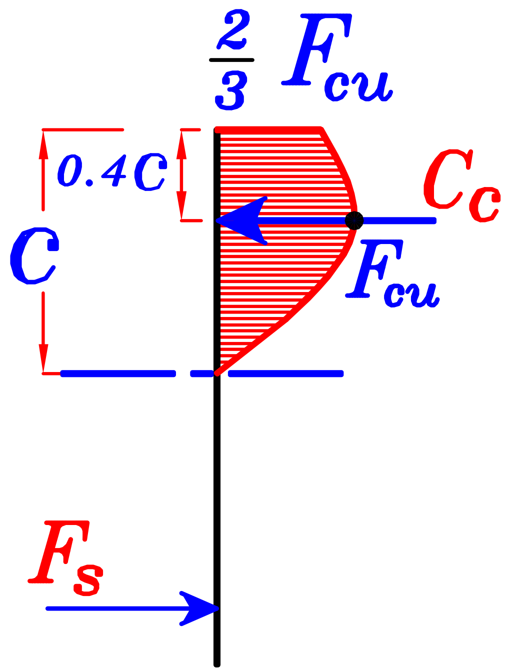
Stress–Strain curve



Strain Diagram



Normal Stress Diagram



Normal Stress Diagram

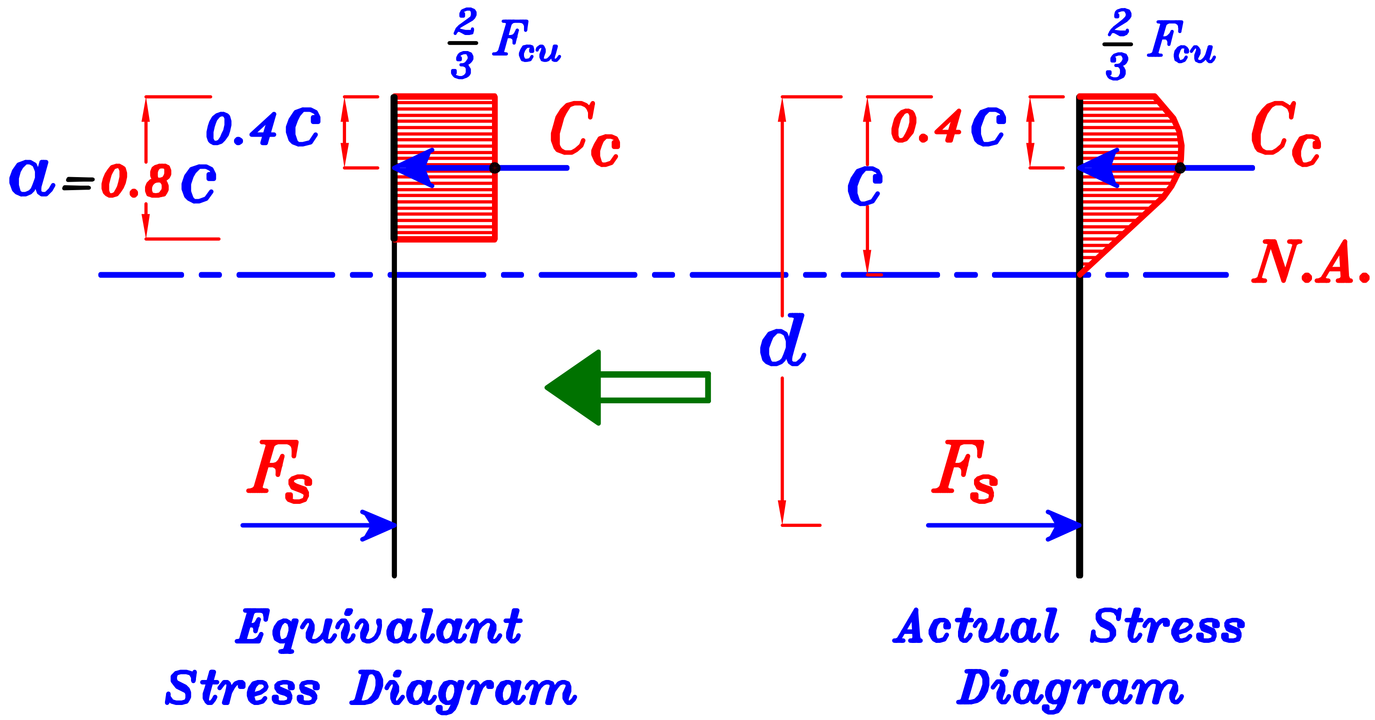
لان شكل ال Stress منحنى لذا فصعب التعامل معه لاننا اذا اردنا حساب مساحه المنحنى أو تحديد مكان المحصله سنحتاج استخدام التكامل .

لذا لتسهيل الحسابات سنلجأ فى الحسابات لـ Stress مكافئ يسمى Equivalent Stress diagram على شكل مستطيل لكى يكون سهل فى الحسابات

و لكن شرط أن تكون مساحته هى نفس مساحه ال Stress الاصلى

و مكان محصلته هو نفس مكان محصله ال Stress الاصلى

محصله القوى C_c تكون لها نفس القيمه و تؤثر فى نفس المكان



لكى تؤثر محصله الـ **Equivalent Stress** تؤثر فى نفس مكان المحصله الاصليه أى على بعد $0.4\,C$

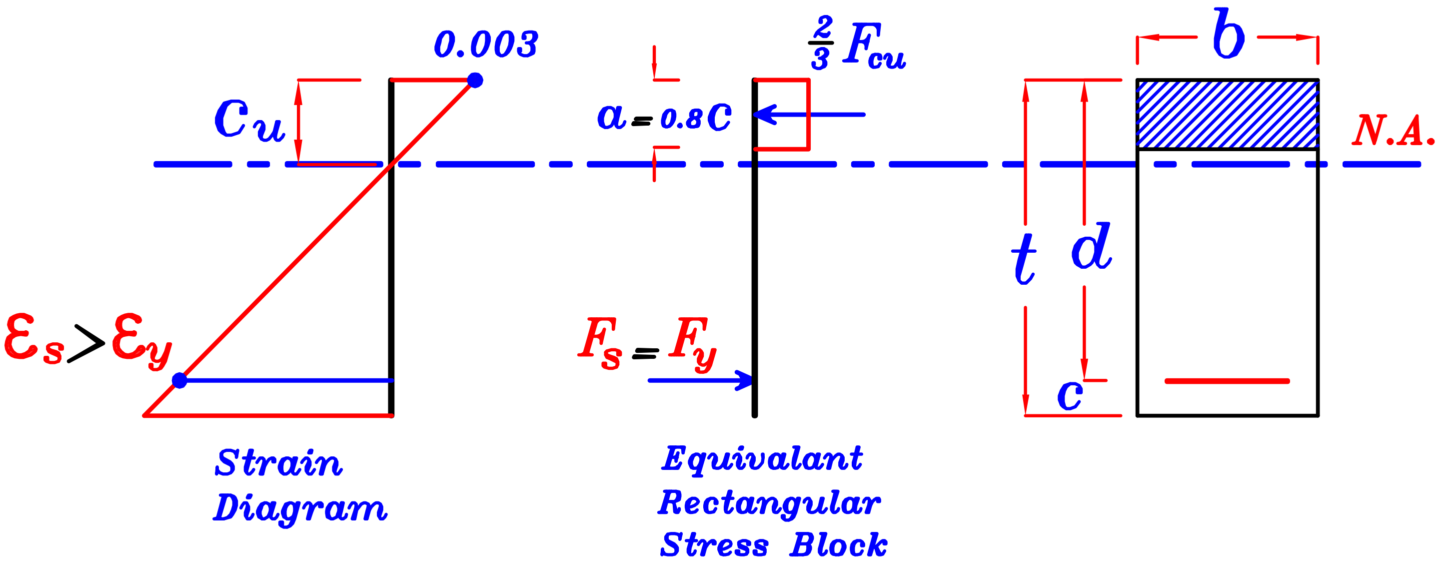
اذا سيكون طول الـ **Equivalent Stress** يساوى a حيث $(a = 0.8\,C)$

و بتساوى مساحه الـ **Equivalent Stress** بمساحه الـ **Stress** الاصلى المحسوب بالتكامل

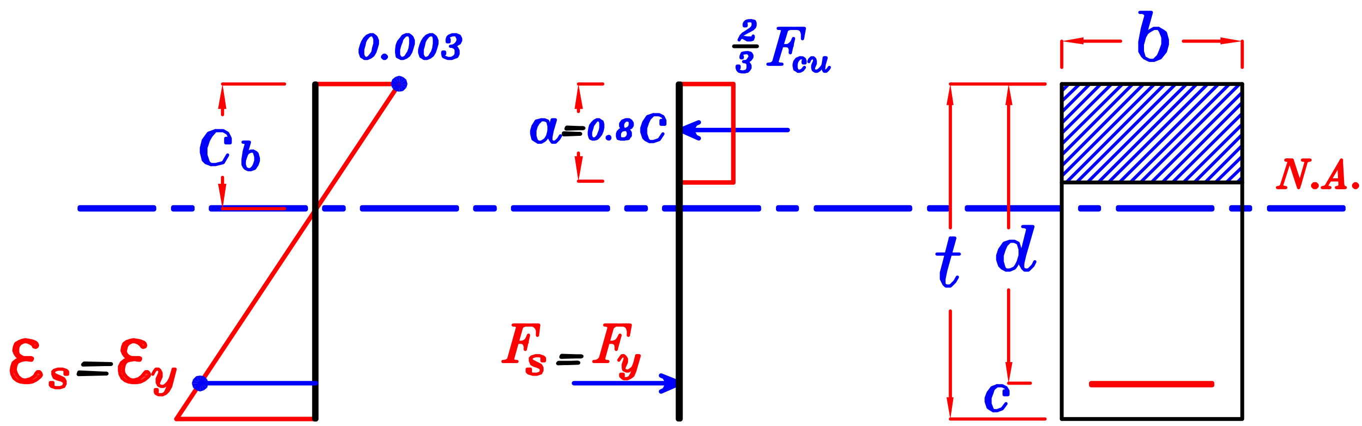
اتضح ان القيمه الثابته لـ **Equivalent Stress** تساوى $\frac{2}{3} F_{cu}$

$\because \boxed{a = 0.8\,C}$ $\qquad$ $\therefore \boxed{C = 1.25\,a}$

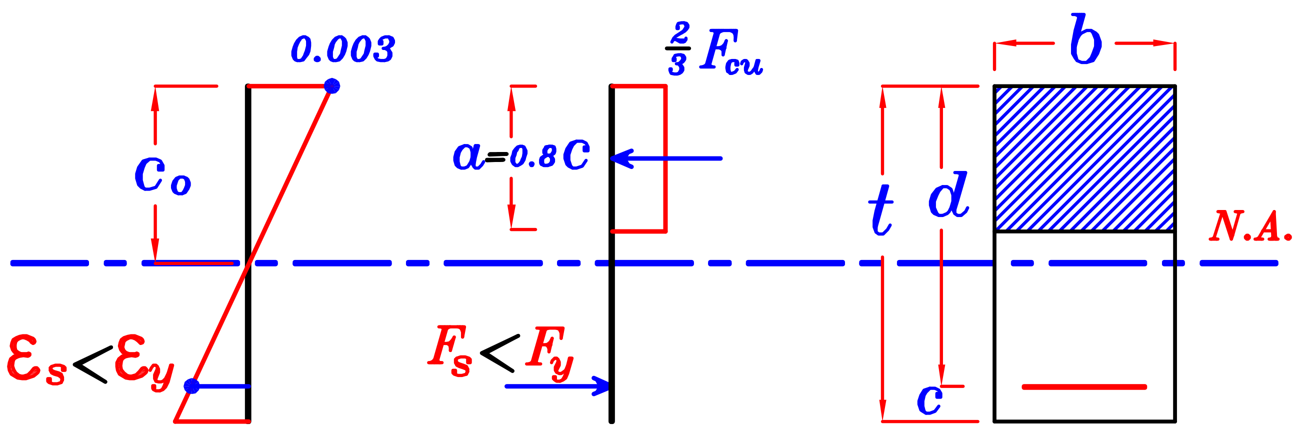
① Under Reinforced Sections.



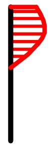
② Balanced Sections.



③ Over Reinforced Sections.



For Beams at Faliure.

فى مرحله الانهيار لان شكل الاصلى لل **stress** عباره عن منحنى 

اذا معادله ال **Normal stress**

$$\boxed{F = \frac{M\,y}{I}}$$

لن تكون صحيحه

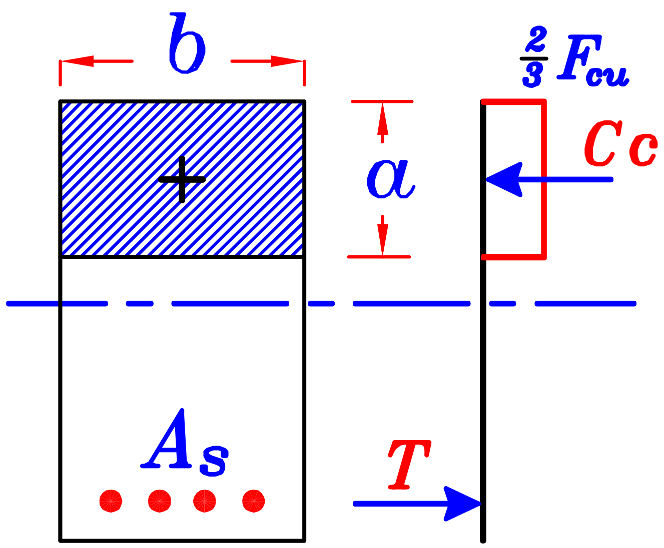
و بالتالى كل حسابات القطاع S_{nv} , I_{nv} , I_g , A_s , n لن تكون صحيحه

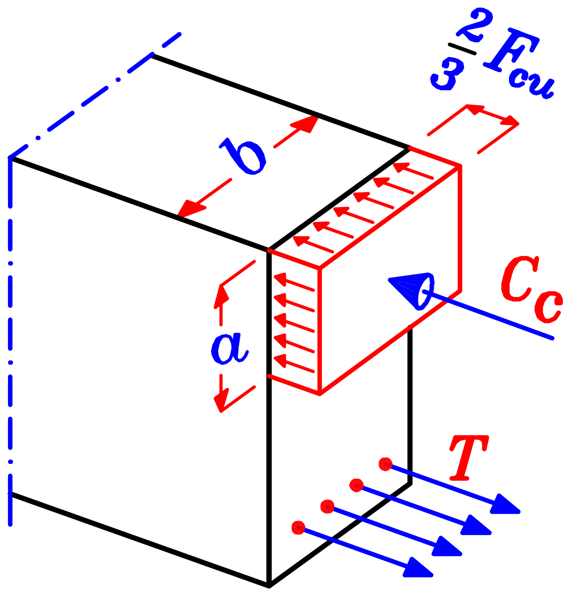
لذا فى مرحله الانهيار لن نستطيع الا استخدام معادلتين فقط.

1. *Equilibrium Equation.* فهم
2. *Compatibility Equation.* حفظ

لحساب قيمه أى قوه تؤثر على القطاع سواء ضغط أو شد

$$\boxed{Force = Stress * Area}$$





Compression on Concrete

$$C_c = Stress * Area = \frac{2}{3}F_{cu} * (a * b)$$

Tension

$$T = Stress * Area = F_s * A_s$$

① Equilibrium Equation. معادله الاتزان

فى أى قطاع لكى يكون متزن يجب أن يكون مجموع القوى الخارجيه تساوى مجموع القوى الداخليه

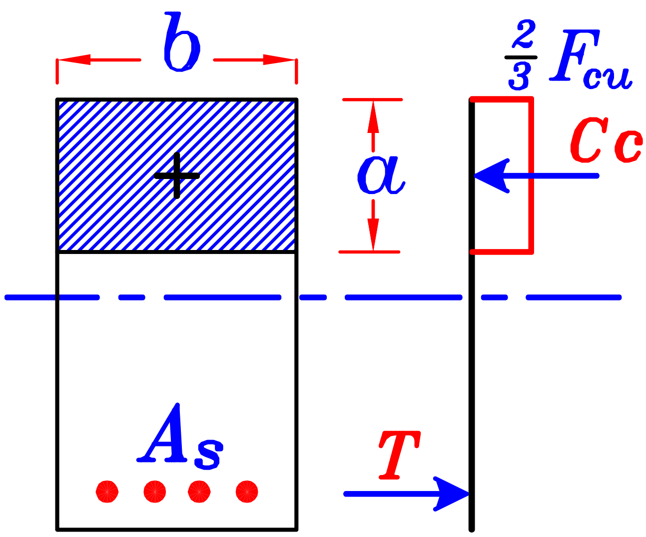
و لان القوى المحوريه الخارجيه على الكمرات تساوى صفر **External Normal Force = Zero**

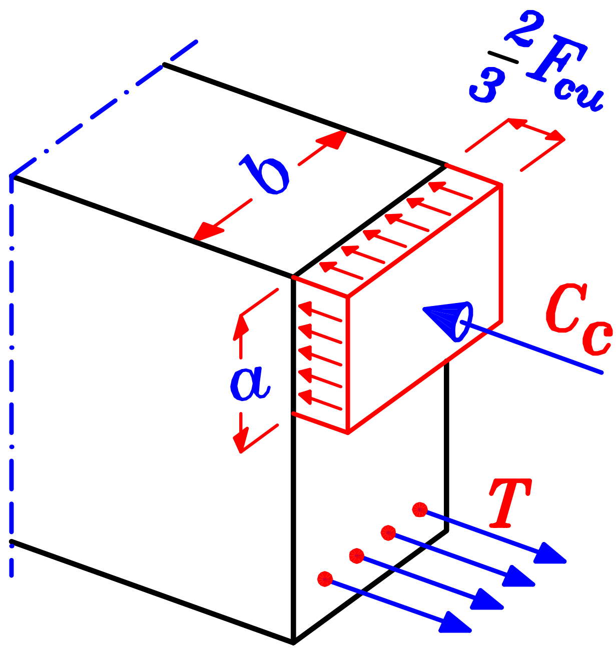
اذا سيكون مجموع القوى المحوريه الداخليه المؤثره على القطاع أيضا تساوى صفر

$\therefore$ Compression Forces + Tension Forces = Zero

$$\therefore \boxed{\text{Compression Forces} = \text{Tension Forces}}$$

ⓐ Without Compression steel.





$$C_c = \text{Stress} * \text{Area} = \frac{2}{3} F_{cu} * (a * b)$$

$$T = \text{Stress} * \text{Area} = F_s * A_s$$

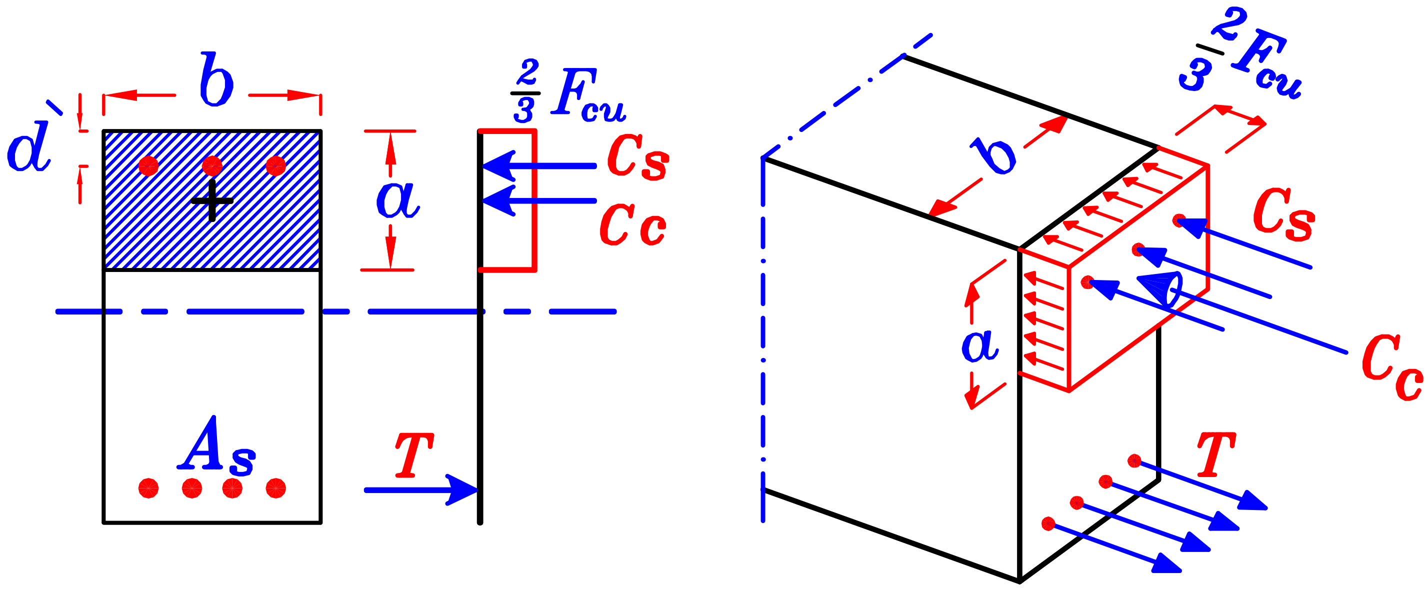
$$\therefore \boxed{\frac{2}{3} F_{cu}\, a\, b = F_s\; A_s}$$

a , F_s مجهولين

For all types of Sections

Under , Balanced & Over

ⓑ With Compression steel.



$$C_c = Stress * Area = \frac{2}{3} F_{cu} * (a * b)$$

Compression on Steel

$$C_s = Stress * Area = F_y * A_{s`}$$

نفرض للتسهيل

$$\boxed{F_{s`} = F_y}$$

$$T = Stress * Area = F_s * A_s$$

$$\therefore \boxed{\frac{2}{3} F_{cu} a\, b + F_y\, A_{s`} = F_s\, A_s}$$

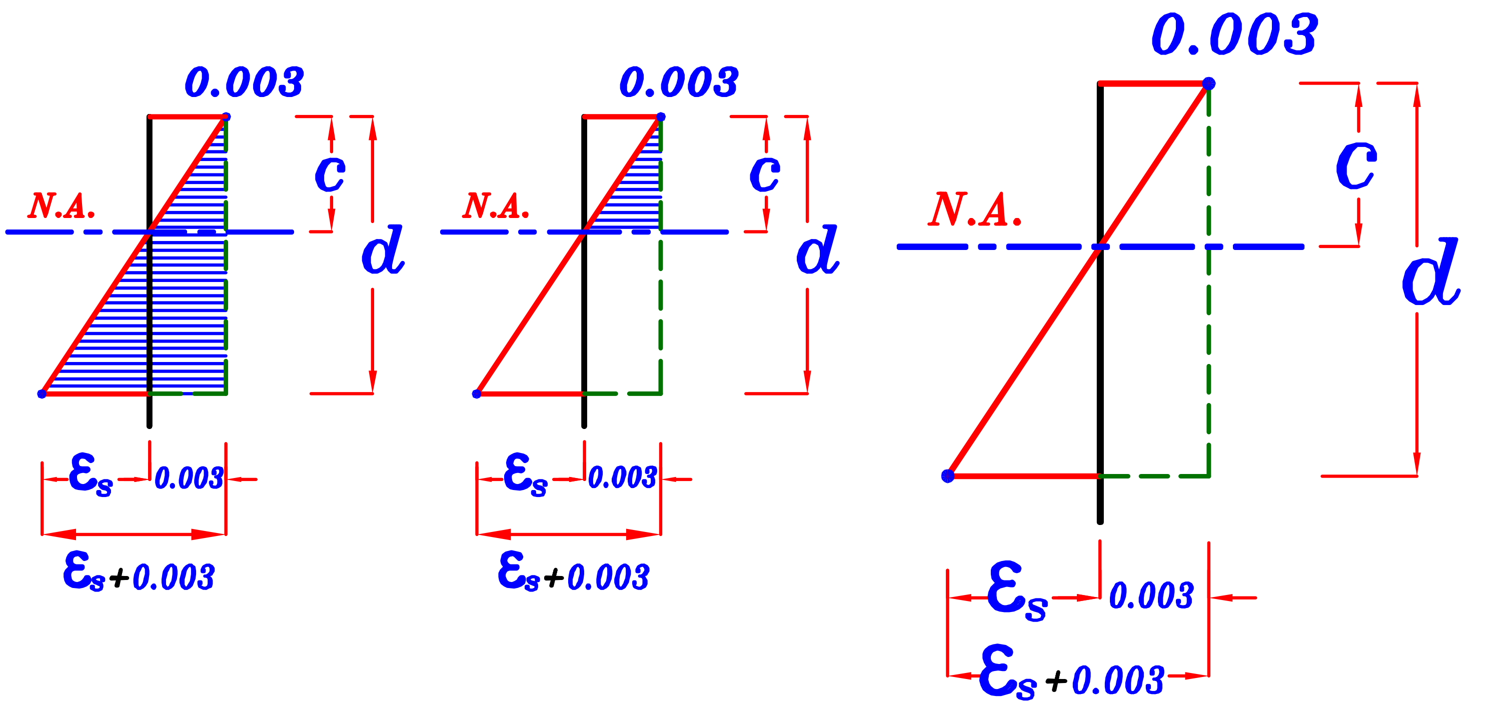
a , F_s مجهولين

For all types of Sections

Under , Balanced & Over

② *Compatibility Equation.* معادله التوافق (التشابه)

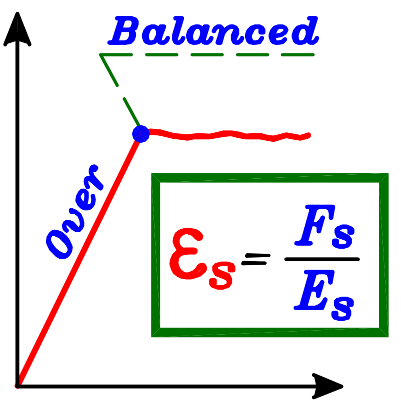
من شكل الـ ***Strain diagram*** يتم عمل تشابه مثلثات



$$\frac{c}{d} = \frac{0.003}{0.003 + \varepsilon_s}$$ من تشابه المثلث

$$\because \varepsilon_s = \frac{F_s}{E_s} = \frac{F_s}{2*10^5}$$

For Balanced & Over only



$$\therefore \frac{c}{d} = \frac{0.003}{0.003 + \frac{F_s}{2*10^5}} = \frac{600}{600 + F_s}$$

$$\boxed{c = 1.25\, a = \frac{600}{600 + F_s} * d}$$

a , F_s مجهولين

Balanced & Over only

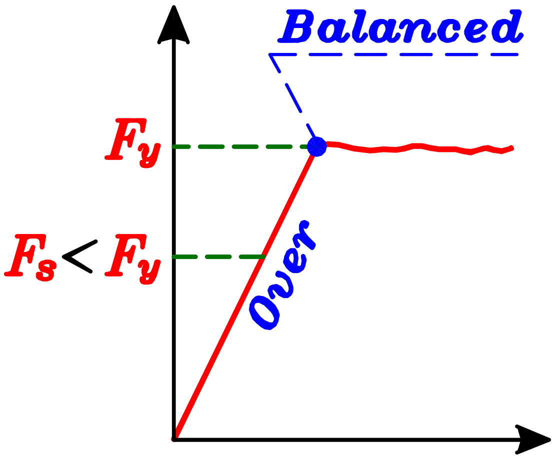
حفظ

Calculation of C_b

From Compatibility Equation.

$$\boxed{C = \frac{600}{600 + F_s} * d}$$

Balanced & Over only



For Balanced section $F_s = F_y$

For Over Reinforced section $F_s < F_y$

$\therefore$ For $C = \dfrac{600}{600 + F_s} * d$

When we take $F_s = F_y$ it will be For Balanced section

$$\therefore \quad \boxed{C_b = \frac{600}{600 + F_y} * d}$$

حفظ

$$\because \quad \boxed{C_u < C_b < C_o}$$

$\therefore$ When $C_u < C_b \longrightarrow$ The section is Under

When $C_u = C_b \longrightarrow$ The section is Balanced

When $C_u > C_b \longrightarrow$ The section is Over

(M_{ult})

Calculation of Ultimate Moment.

هو عزم الإنهيار . أى هو العزم الذى يصل فيه أياً من الحديد
أو الخرسانه إلى الـ *max stress or max strain.*

max. stress (Concrete) $= F_{cu}$

max. stress (Steel) $= F_y$

max. strain (Concrete) $= \varepsilon_c = 0.003$

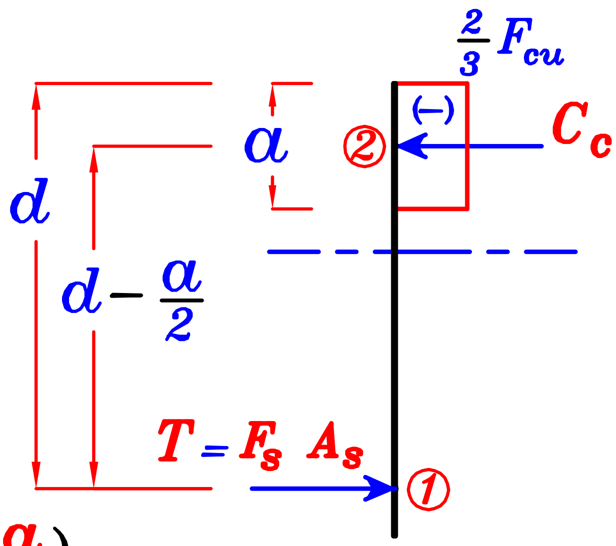
max. strain (Steel) $= \varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

Note $\boxed{\text{When } \varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y}$

How to Determine M_{ult} For a known Section.

$$C_c = \frac{2}{3} F_{cu} * a * b$$

$$T = F_s * A_s$$



M_{ult} = M at point ①

= M at point ②

M_{ult} at point ① $= C_c \, (d - \frac{a}{2})$

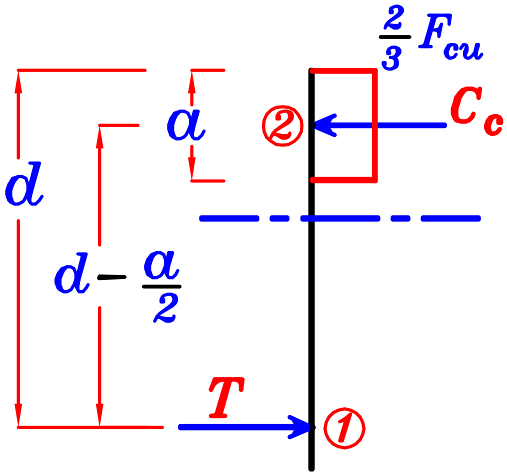
$= \frac{2}{3} F_{cu} \, a \, b \, (d - \frac{a}{2})$

M_{ult} at point ② $= T \, (d - \frac{a}{2}) = F_s * A_s \, (d - \frac{a}{2})$

But a , F_s ?? $\therefore$ We have to get ((a , F_s)) First.

To Calculate M_{ult}

① With Tension Steel only.



① Get $C_b = \dfrac{600}{600+F_y} * d$

② Use equilibrium equation. $C_c = T$

$$\frac{2}{3}F_{cu} * (a*b) = A_s * F_s \text{ --- } a, F_s = ??$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}F_{cu} * (a*b) = A_s * F_y \longrightarrow \text{Get } a \longrightarrow \text{Get } C = 1.25\,a$$

③ Check C

* IF $C \leqslant C_b \longrightarrow$ The Section is Under Reinforced or Balanced Sec. and the assumption is right $F_s = F_y$

④ $\therefore$ $\boxed{M_{ult} = \frac{2}{3}F_{cu}\,a\,b\left(d-\frac{a}{2}\right) = A_s F_y \left(d-\frac{a}{2}\right)}$

* IF $C > C_b \longrightarrow$ The Section is Over Reinforced Sec. and the assumption is wrong $F_s \neq F_y$

$\therefore$ To get the right value oF a, F_s

① From equilibrium eqn.

$$\frac{2}{3}F_{cu}\,a\,b = A_s\,F_s \text{ ----- ① } \quad a = ?,\ F_s = ?$$

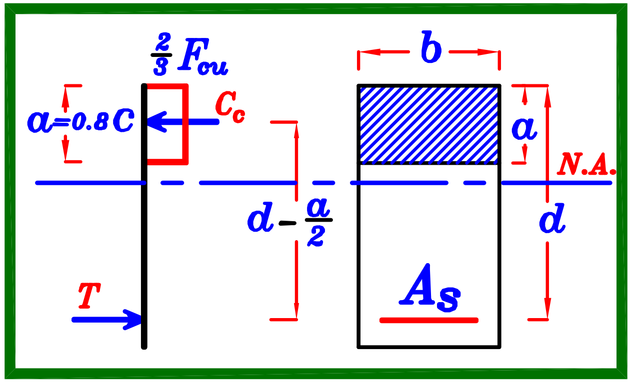
② From compatibility eqn.

$$C = 1.25\,a = \frac{600}{600+F_s} * d \text{ ----- ② } \quad a = ?,\ F_s = ?$$

From eqns. ①, ② Get a, F_s

④ $\therefore$ $\boxed{M_{ult} = \frac{2}{3}F_{cu}\,a\,b\left(d-\frac{a}{2}\right) = A_s F_s \left(d-\frac{a}{2}\right)}$

Calculation of M_{ult} For R-sec. (With Ten. Steel only)



Get $C_b = \frac{600}{600+F_y} * d$

From equilibrium eqn. $\frac{2}{3}F_{cu} * (a * b) = F_s * A_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (The section is under reinforced or Balanced Sec.)

$\therefore F_s = F_y$ $\quad \therefore \frac{2}{3}F_{cu} * (a * b) = F_y * A_s \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\, a$

IF $\boxed{C}$

IF $C \leqslant C_b$

- $C < C_b$ — Under Reinforced Section
- $C = C_b$ — Balanced Section

and the assumption is right $F_s = F_y$

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = A_s F_y \left(d - \frac{a}{2}\right)}$$

IF $C > C_b$

Over Reinforced Section

and the assumption is wrong $F_s \neq F_y$

To get the right value of a, F_s

$\frac{2}{3}F_{cu}\, a\, b = F_s A_s$ ---- ① $\quad a = ?\,, \; F_s = ?$

$C = 1.25\, a = \frac{600}{600+F_s} * d$ ---- ② $\quad a = ?\,, \; F_s = ?$

From eqns. ①, ② Get a, F_s

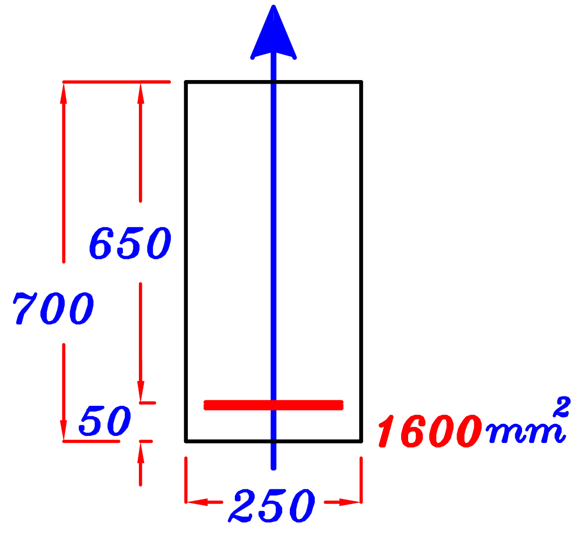
$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) = F_s A_s \left(d - \frac{a}{2}\right)}$$

Example.

Data.

$F_{cu} = 25 \ N \backslash mm^2$

st. 360/520



Req.

For the shown Cross-Section

1- Calculate $M_{ult.}$

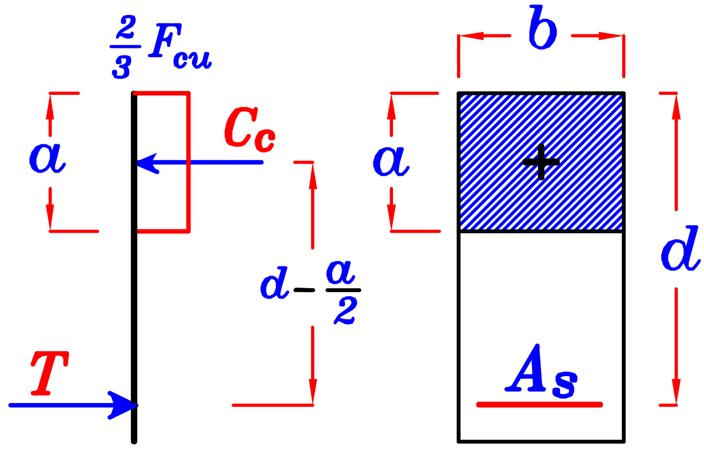
2- Determine which type of Failure will occur For that section.

Solution.

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \ mm$$

$$C_c = Stress * Area = \frac{2}{3} F_{cu} * a * b$$

$$T = Stress * Area = F_s * A_s$$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

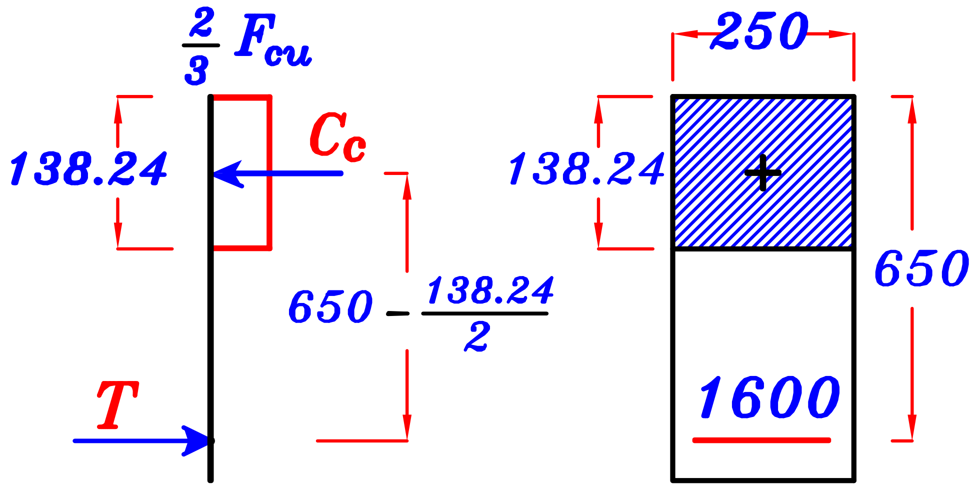
Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (1600)(360) \longrightarrow a = 138.24 \ mm$$

③ $\therefore\ C = 1.25\ a = 1.25 * 138.24 = 172.8\ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$



④ By taking the moment about the steel.

$$\therefore\ M_{ult} = C_c * \left(d - \frac{a}{2}\right) = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(138.24)(250)\left(650 - \frac{138.24}{2}\right)$$

$$= 334586880\ N.mm = 334.5\ kN.m$$

④ OR By taking the moment about concrete.

$$M_{ult} = T * \left(d - \frac{a}{2}\right) = F_y * A_s \left(d - \frac{a}{2}\right)$$

$$= (360 * 1600)\left(650 - \frac{138.24}{2}\right) = 334586880\ N.mm$$

$$= 334.5\ kN.m$$

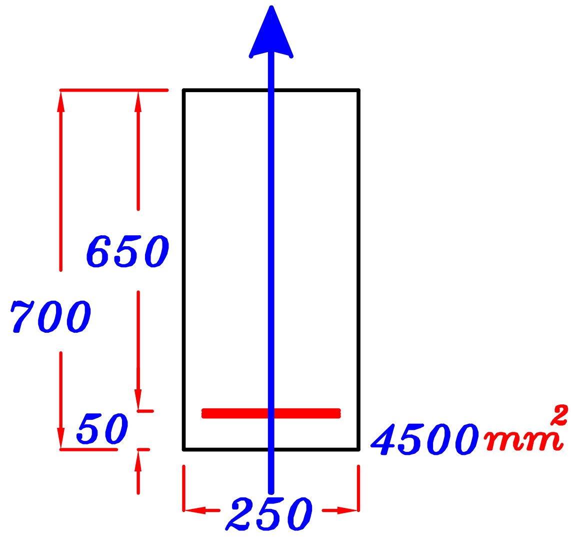
$$\therefore\ \boxed{M_{ult} = 334.5\ kN.m}$$

Example.

Data.

$F_{cu} = 25 \quad N\backslash mm^2$

st. 360/520



Req.

For the shown Cross-Section

1- Calculate $M_{ult.}$

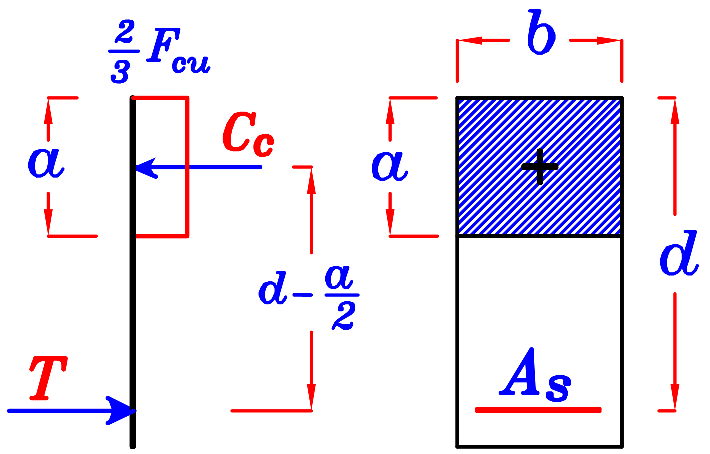
2- Determine which type of Failure will occur For that section.

Solution.

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \text{ mm}$$

$$C_c = \text{Stress} * \text{Area} = \frac{2}{3} F_{cu} * a * b$$

$$T = \text{Stress} * \text{Area} = F_s * A_s$$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (4500)(360) \longrightarrow a = 388.8 \text{ mm}$$

③ $\therefore\ C = 1.25\,a = 1.25 * 388.8 = 486.0\ mm > C_b$

$\therefore$ $\boxed{\text{The Section is Over Reinforced Sec.}}$

and the assumption is wrong $F_s < F$

To get the right value of a, F_s

$$\therefore\ \frac{2}{3} F_{cu}\ a\ b = A_s\ F_s$$

$$\therefore\ \frac{2}{3}(25)(a)(250) = (4500)(F_s)$$

$$\therefore\ F_s = 0.926\ a \quad \text{---- ①} \quad a = ?\ ,\ F_s = ?$$

$$\because\ C = 1.25\,a = \frac{600}{600 + F_s} * d \quad \text{---- ②} \quad a = ?\ ,\ F_s = ?$$

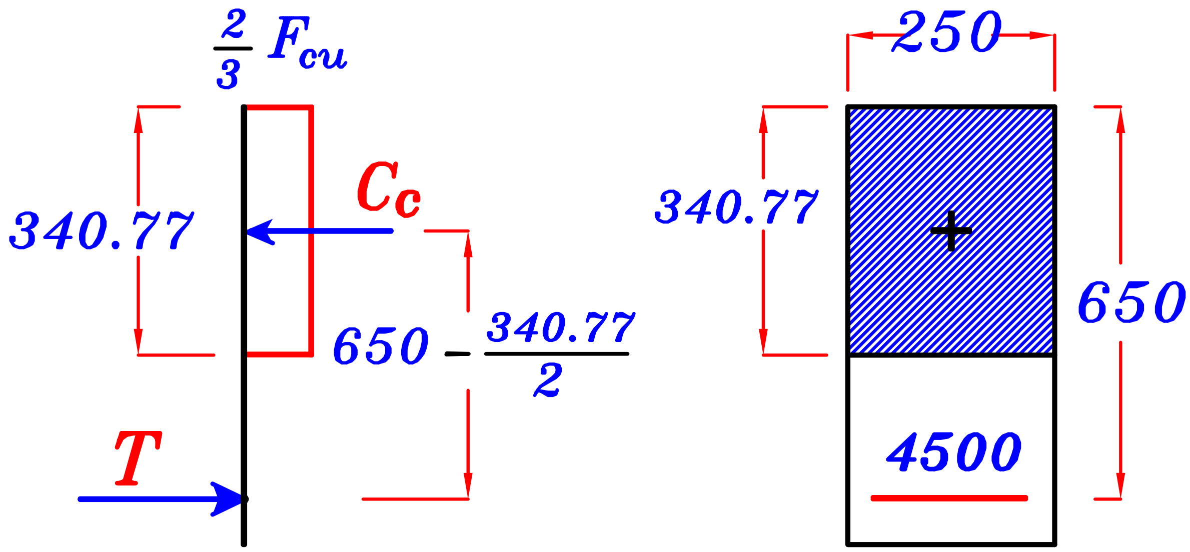
From eqns. ①, ② Get a, F_s

$$\therefore\ 1.25\,a = \frac{600}{600 + 0.926\ a} * 650$$

$$\therefore\ \boxed{a = 340.77\ mm}$$

$$F_s = 0.926\,(340.77) = 315.5\ N\backslash mm^2$$

$$\boxed{F_s = 315.5\ N\backslash mm^2} < F_y$$



④ By taking the moment about the steel.

$$\therefore \; M_{ult} = C_c * \left(d-\frac{a}{2}\right) = \frac{2}{3} F_{cu}\, a\, b \left(d-\frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(340.77)(250)\left(650-\frac{340.77}{2}\right)$$

$$= 680993348.1 \text{ N.mm} = 680.99 \text{ kN.m}$$

④ OR By taking the moment about concrete.

$$M_{ult} = T * \left(d-\frac{a}{2}\right) = F_s * A_s \left(d-\frac{a}{2}\right)$$

$$=(315.5*4500)\left(650-\frac{340.77}{2}\right) = 680933396.3 \text{ N.mm}$$

$$= 680.93 \text{ kN.m}$$

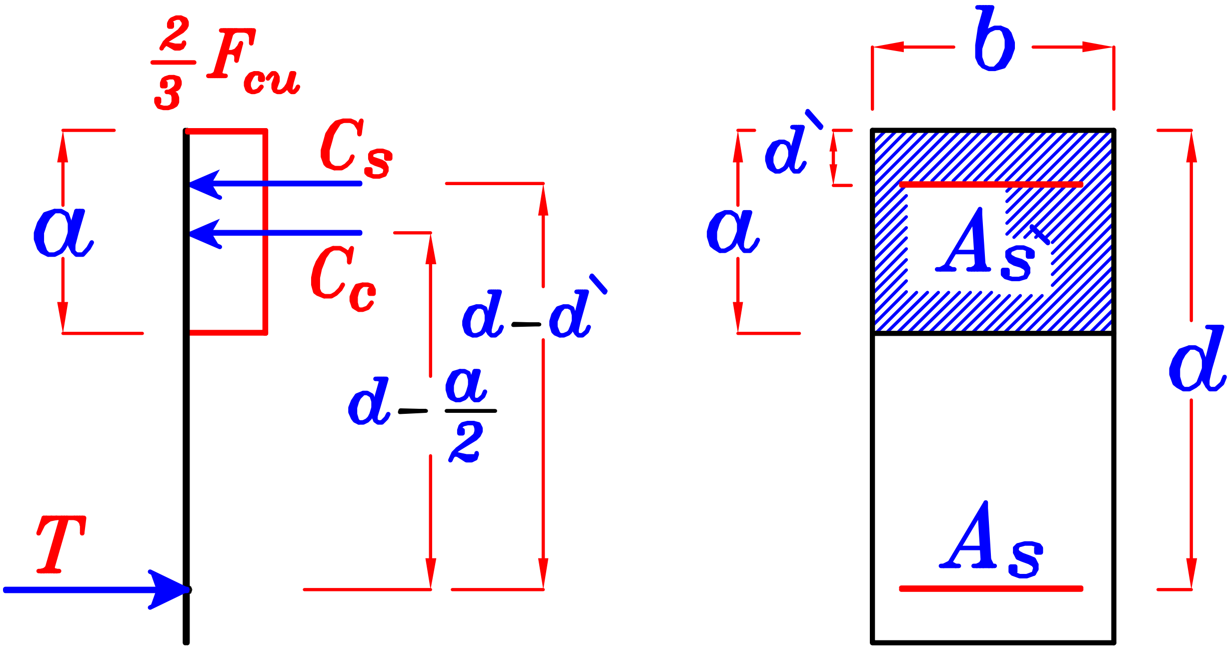
$$\therefore \; \boxed{M_{ult} = 680.93 \text{ kN.m}}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

عند حساب M_{ult} وكان هناك حديد جهه الضغط $(A_{s`})$

نعمل حل تقريبى للتسهيل بأن نعتبر $F_{s`} = F_y$

و لحساب الـ M_{ult} مع وجود $(A_{s`})$ بدقه سنذكرها فى أخر الملف Page No. 103



$$C_c = Stress * Area = \frac{2}{3} F_{cu} * (a\ b)$$

$$C_s = Stress * Area = F_y * A_{s`}$$

By taking the moment about the steel.

$$\boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + F_y * A_{s`} (d - d`)}$$

Example.

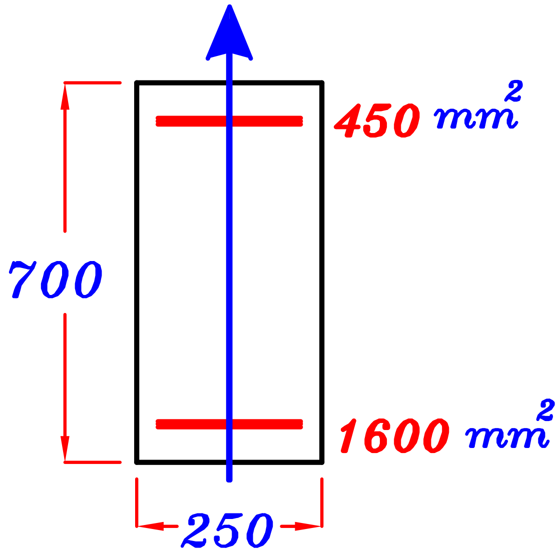
Data.

$F_{cu} = 25 \ N \backslash mm^2$

st. 360/520

Req.

For the shown Cross-Section



1_ Calculate $M_{ult.}$

2_ Determine which type of Failure will occur For that section.

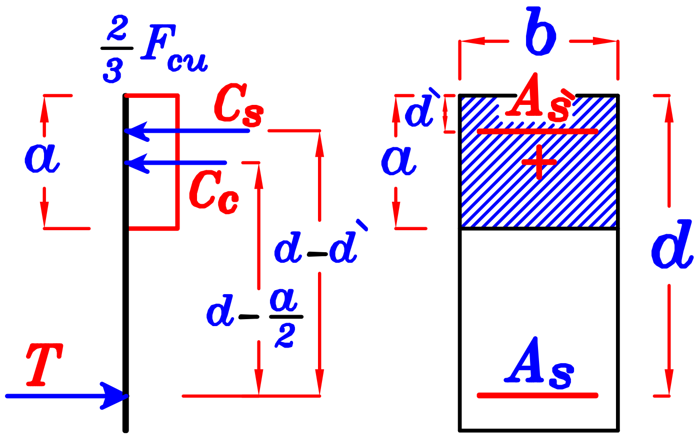
Solution. $\because \frac{A_s`}{A_s} = \frac{450}{1600} = 0.28 > 0.2 \quad \therefore$ Use $A_s`$

① $C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \ mm$

$C_c = Stress * Area = \frac{2}{3} F_{cu} * a * b$

$C_s = Stress * Area = F_y * A_s`$

$T = Stress * Area = F_s * A_s$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b + F_y * A_s` = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

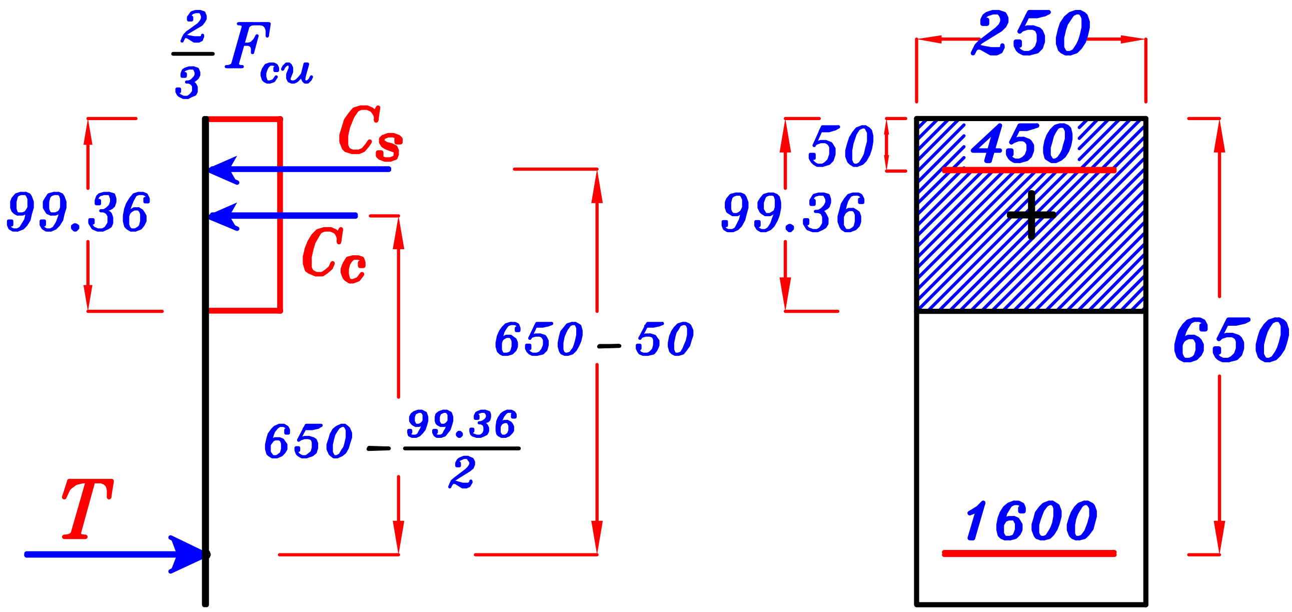
$$\frac{2}{3} (25)(a)(250) + (360)(450) = (360)(1600)$$

$$\boxed{a = 99.36 \ mm}$$

③ $\therefore C = 1.25\, a = 124.2 \text{ mm} < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$



④ By taking the moment about the steel.

$$M_{ult} = C_c * \left(d - \frac{a}{2}\right) + C_s * (d - d`)$$

$$M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + F_y * A_{s`} (d - d`)$$

$$M_{ult} = \frac{2}{3}(25)(99.36)(250)\left(650 - \frac{99.36}{2}\right) + 360 * 450\,(650 - 50)$$

$$= 345732480 \text{ N.mm} = 345.7 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 345.7 \text{ kN.m}}$$

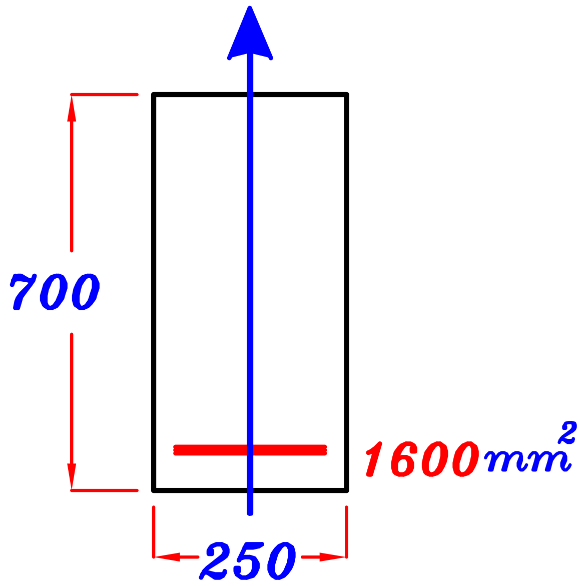
Note.

فى حاله وجود حديد جهه ال $(A_{s`})$

فان الزياده الحادثه فى قيمه M_{ult} لن تكون كبيره

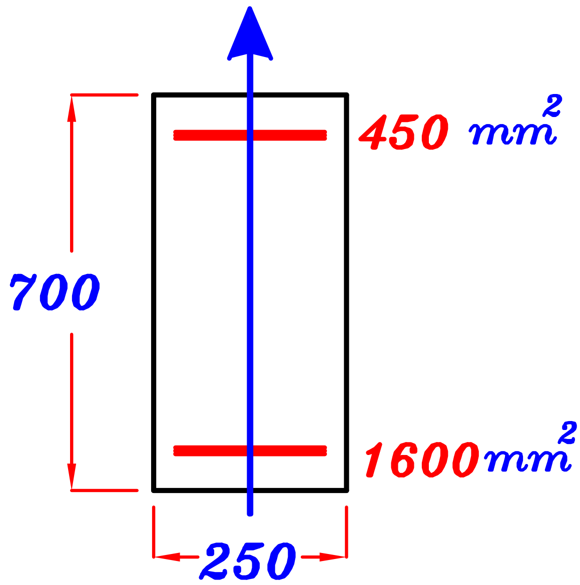
Example Page 61

$$\boxed{M_{ult} = 334.5 \text{ kN.m}}$$



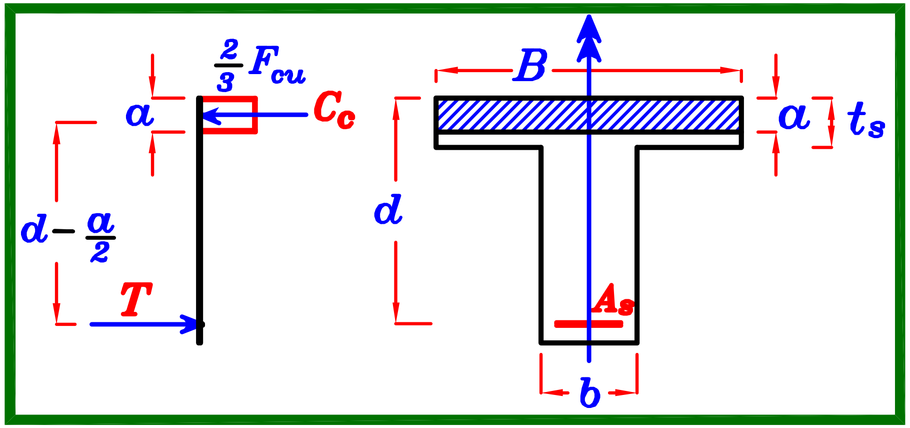
Example Page 67

$$\boxed{M_{ult} = 345.7 \text{ kN.m}}$$



Calculation of M_{ult} For T-sec.
(With Ten. Steel only)

Get $C_b = \dfrac{600}{600 + F_y} * d$



Assume $a \leqslant t_s$

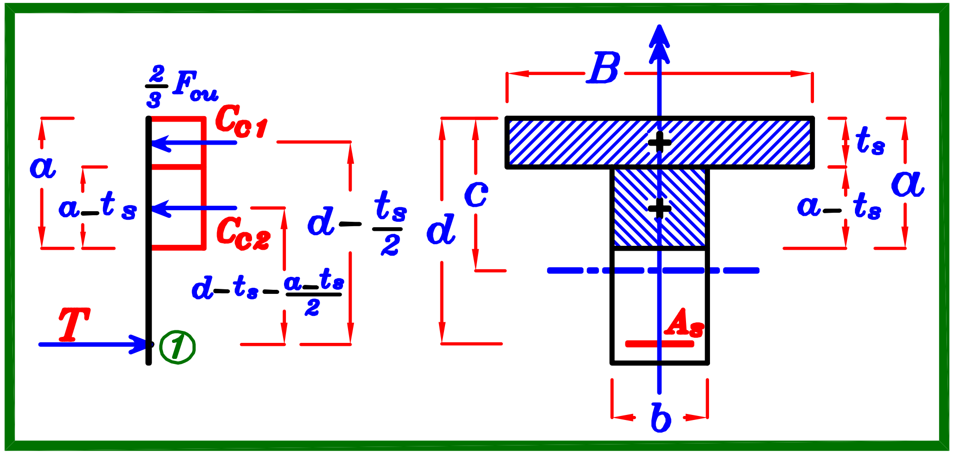
From equilibrium eqn. $\frac{2}{3} F_{cu} * (a * B) = F_s * A_s$

assume $F_s = F_y$ (The section is under reinforced or Balanced Sec.)

$\therefore \frac{2}{3} F_{cu} * (a * B) = F_y * A_s \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\, a$

IF $a \leqslant t_s \longrightarrow C < C_b$ the First & second assumptions are right.

$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a B \left(d - \frac{a}{2}\right) = F_y\, A_s \left(d - \frac{a}{2}\right)}$$



IF $a > t_s$ The First assumption is wrong.

From equilibrium eqn. $C_{c1} + C_{c2} = T$

$\frac{2}{3} F_{cu} * t_s * B + \frac{2}{3} F_{cu} * (a - t_s) * b = F_s * A_s$

assume $F_s = F_y$ (The section is under reinforced or Balanced Sec.)

Get $a > t_s \longrightarrow$ Get $C = 1.25\, a$

IF $\boxed{C}$

IF $C \leqslant C_b$ right assumption

$M_{ult} = C_{c1}\left(d - \frac{t_s}{2}\right) + C_{c2}\left(d - t_s - \frac{a - t_s}{2}\right)$

$$\boxed{M_{ult} = \left(\frac{2}{3} F_{cu} * t_s * B\right)\left(d - \frac{t_s}{2}\right) + \left(\frac{2}{3} F_{cu} * (a - t_s) * b\right)\left(d - t_s - \frac{a - t_s}{2}\right)}$$

IF $C > C_b$ wrong assumption

To get the right value of a, F_s

$\frac{2}{3} F_{cu} * t_s * B + \frac{2}{3} F_{cu} * (a - t_s) * b = A_s * F_s$ --- ① $a = ?$, $F_s = ?$

$C = 1.25\, a = \dfrac{600}{600 + F_s} * d$ --- ② $a = ?$, $F_s = ?$

From eqns. ① , ② Get a , F_s

$$\boxed{M_{ult} = \left(\frac{2}{3} F_{cu} * t_s * B\right)\left(d - \frac{t_s}{2}\right) + \left(\frac{2}{3} F_{cu} * (a - t_s) * b\right)\left(d - t_s - \frac{a - t_s}{2}\right)}$$

Example.

Data.

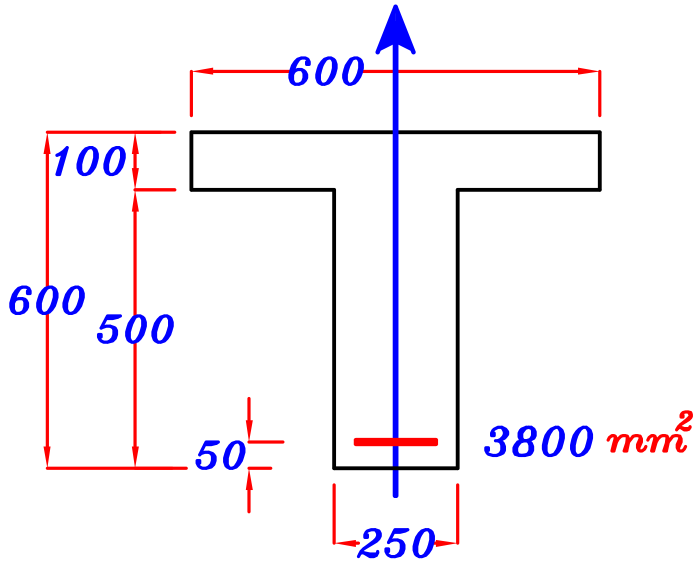
$F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

Req.

For the shown Cross-Section

Calculate $M_{ult.}$

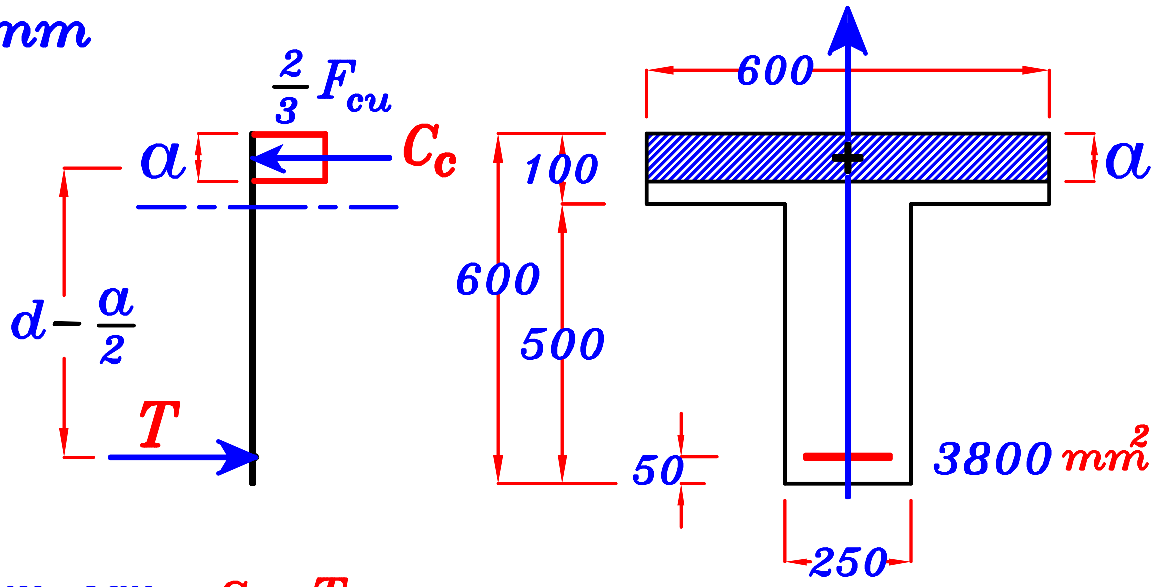


Solution.

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 550 = 343.75 \text{ mm}$

② Assume $a \leqslant t_s$

$a < 100 \text{ mm}$



From equilibrium eqn. $C_c = T$

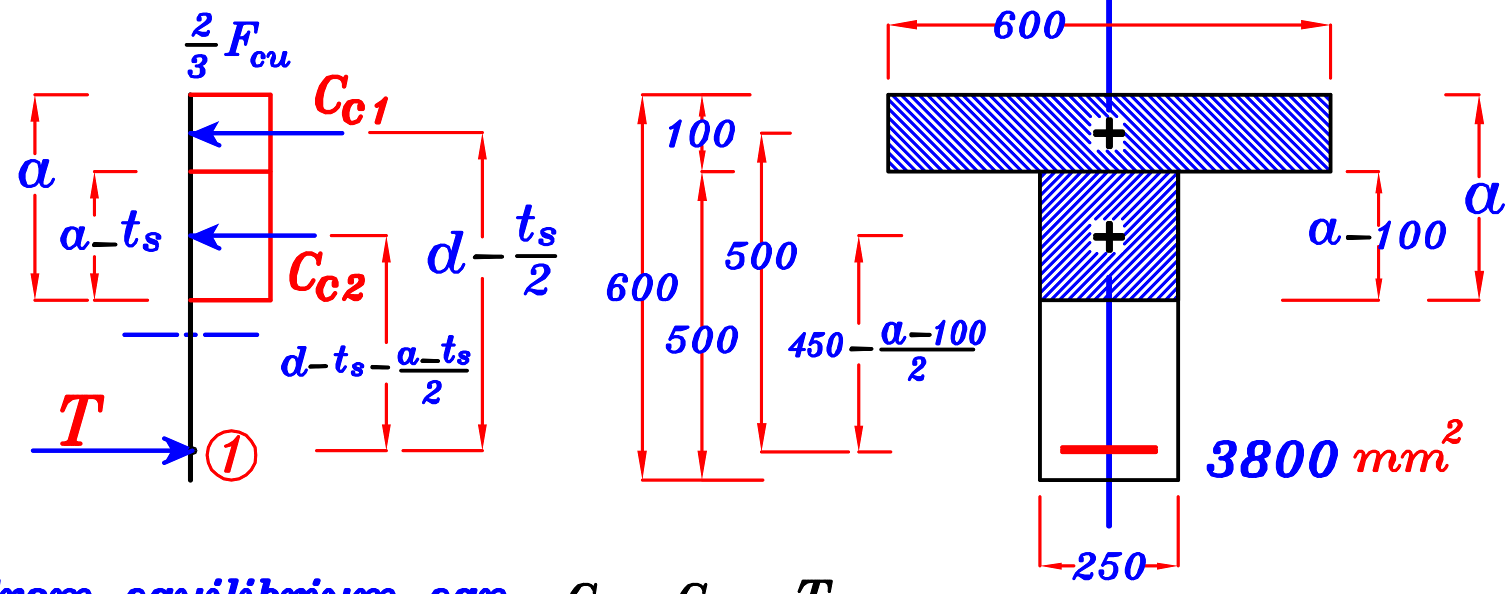
$$\frac{2}{3} F_{cu} * a * B = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(600) = (360)(3800) \longrightarrow a = 136.8 \text{ mm} > t_s$$

$a > t_F$ wrong assumption $\therefore$ Take $a > t_s$

$\therefore$ **Take** $a > t_F$



From equilibrium eqn. $C_{c1} + C_{c2} = T$

$$\frac{2}{3}F_{cu} * t_s * B + \frac{2}{3}F_{cu} * (a - t_s) * b = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (**under reinforced or Balanced Sec.**)

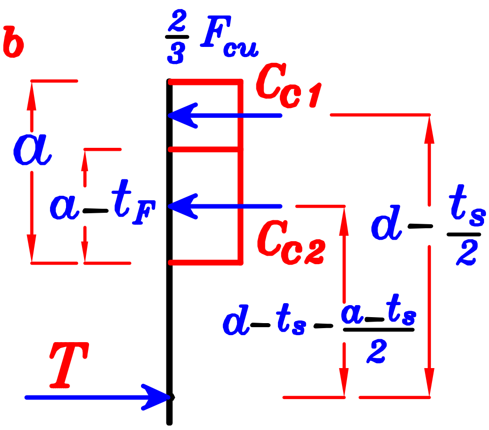
$$\frac{2}{3}(25)(100)(600) + \frac{2}{3}(25)(a - 100)(250) = (3800)(360)$$

$\longrightarrow a = 188.32\ mm > t_s$ **right assumption**

$\therefore C = 1.25\ a = 1.25 * 188.32 = 235.4\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$



$$M_{ult} = C_{c1}\left(d - \frac{t_s}{2}\right) + C_{c2}\left(d - t_s - \frac{a - t_s}{2}\right)$$

$$M_{ult} = \left(\frac{2}{3}F_{cu} * t_s * B\right)\left(d - \frac{t_s}{2}\right) + \left(\frac{2}{3}F_{cu} * (a - t_s) * b\right)\left(d - t_s - \frac{a - t_s}{2}\right)$$

$$= \frac{2}{3}(25)(100)(600)\left(550 - \frac{100}{2}\right) + \frac{2}{3}(25)(188.32 - 100)(250)\left(550 - 100 - \frac{188.32 - 100}{2}\right)$$

$$= 649349120\ N.mm = 649.34\ kN.m$$

$$\therefore \boxed{M_{ult} = 649.34\ kN.m}$$

Example.

Data.

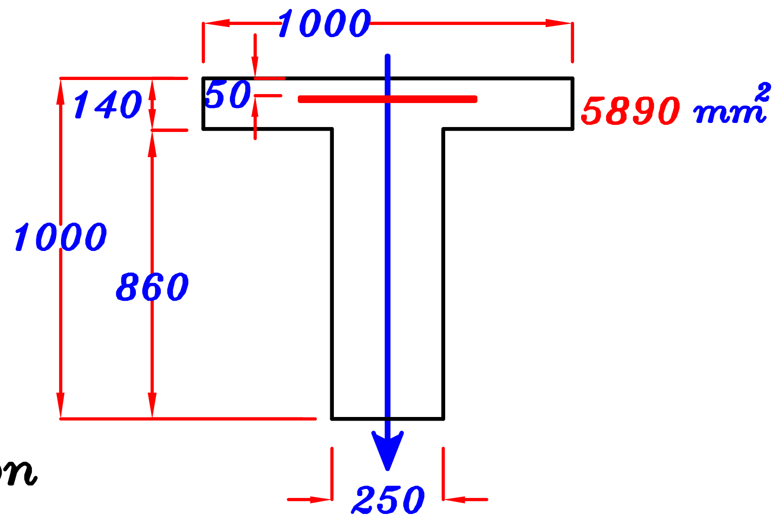
$F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

Req.

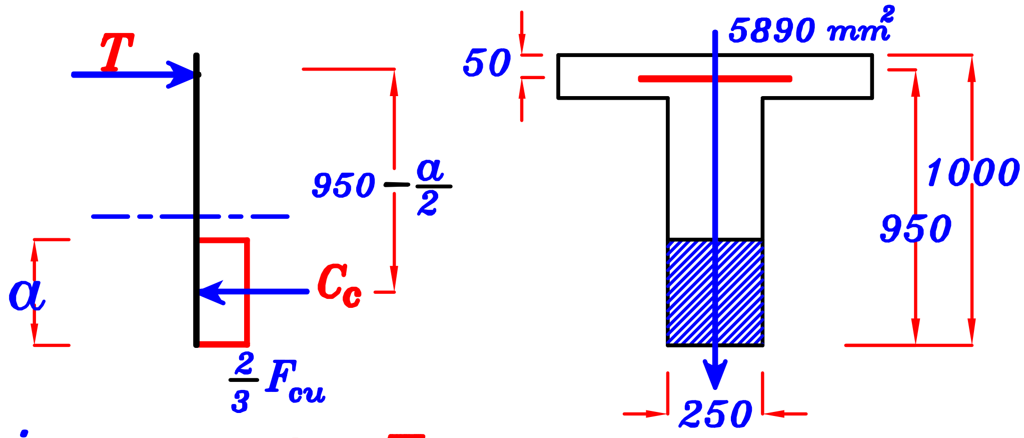
For the shown Cross-Section

Calculate $M_{ult.}$



Solution.

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 950 = 593.75 \ mm$



② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(250) = (360)(5890) \longrightarrow a = 508.9 \ mm$$

$$\therefore C = 1.25 \ a = 1.25 * 508.9 = 636.1 \ mm > C_b$$

$\therefore$ **The Section is Over Reinforced Sec.**

and the assumption is wrong $F_s < F_y$

To get the right value of a, F_s

$\therefore \frac{2}{3} F_{cu} \ a \ b = F_s \ A_s \qquad \therefore \frac{2}{3} (25)(a)(250) = (F_s)(5890)$

$\therefore F_s = 0.707 \ a$ --- ① $a = ?, \ F_s = ?$

$$\because \quad C = 1.25\,a = \frac{600}{600 + F_s} * d \quad --- \; ② \qquad a = ? \;,\; F_s = ?$$

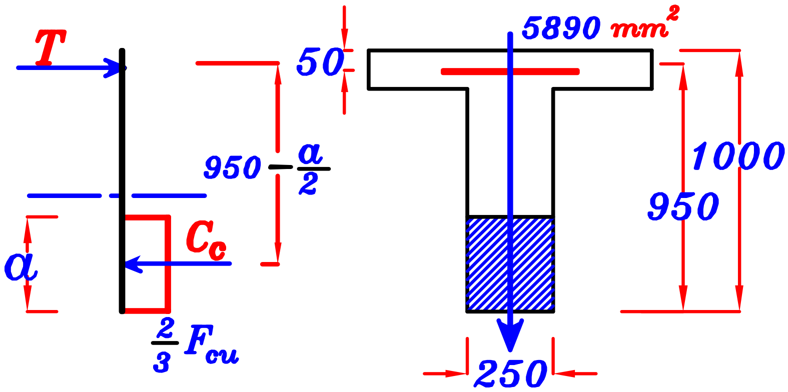
From eqns. ①, ② Get a, F_s

$$\therefore \quad 1.25\,a = \frac{600}{600 + 0.707\,a} * 950$$

$$\therefore \quad \boxed{a = 483.98 \ mm}$$

$$F_s = 0.707 \, (483.98) = 342.17 \ N/mm^2$$

$$\boxed{F_s = 342.17 \ N \backslash mm^2} < F_y$$



$$\therefore \quad M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(483.98)(250)\left(950 - \frac{483.98}{2}\right) = 1427761166 \ N.mm$$

$$= 1427.76 \ kN.m$$

$$\therefore \quad \boxed{M_{ult} = 1427.76 \ kN.m}$$

or

$$M_{ult} = A_s F_s \left(d - \frac{a}{2}\right)$$

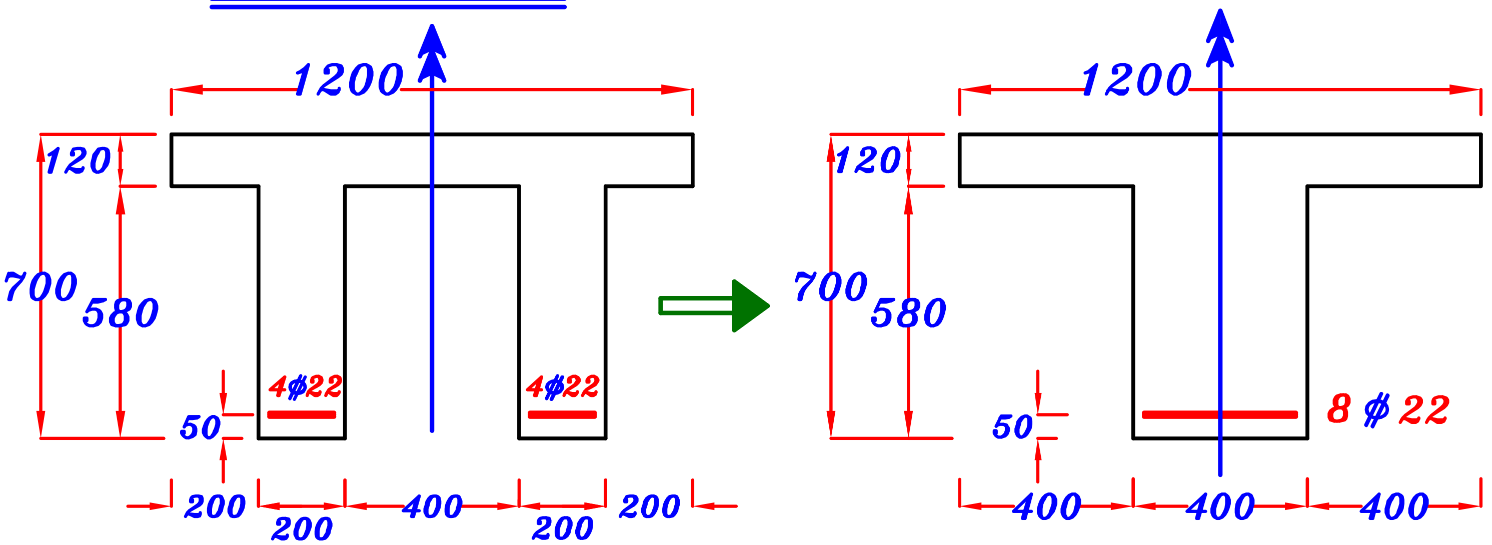
$$M_{ult} = (5890)(342.17)\left(950 - \frac{483.98}{2}\right) = 1426910114 \ N.mm$$

$$1426.91 \ kN.m$$

$$\therefore \quad \boxed{M_{ult} = 1426.91 \ kN.m}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

Example.



Data. $F_{cu} = 25 \; N\backslash mm^2$ st. 360/520

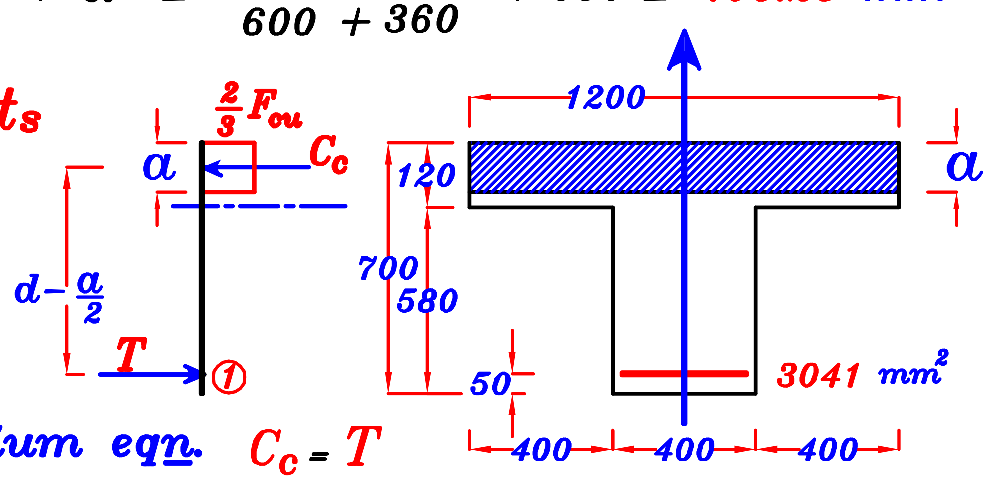
Req. For the shown Cross-Section Calculate Factor of Safty.

Solution. $A_s = 8\phi 22 = 8\left[\frac{\pi * 22^2}{4}\right] = 3041 \; mm^2$

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 650 = 406.25 \; mm$$

② Assume $a \leqslant t_s$

$a < 120 \; mm$



③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(1200) = (3041)(360) \longrightarrow a = 54.74 \; mm < t_s \quad \therefore \; 0.K.$$

$$\therefore \; C = 1.25 \; a = 1.25 * 54.74 = 68.42 \; mm < C_b$$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$ and the assumption is right $F_s = F_y$

$$\therefore \; M_{ult} = \frac{2}{3} F_{cu} \, a \, B \left(d - \frac{a}{2}\right)$$

$$M_{ult} = \frac{2}{3}(25)(54.74)(1200)\left(650 - \frac{54.74}{2}\right) = 681655324 \; N.mm = 681.65 \; kN.m$$

$$\therefore \; \boxed{M_{ult} = 681.65 \; kN.m}$$

Calculate M_w

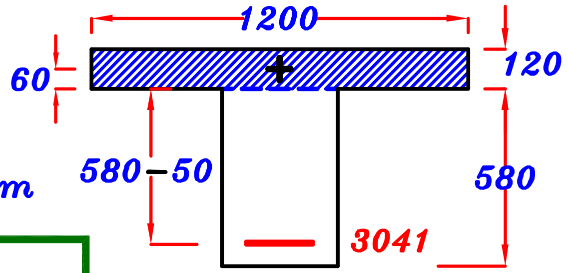
$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_{cb} = 9.5 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

$S_{nv.}(above) = 120 * 1200 * (60) = 8640000 \quad mm^3$

$S_{nv.}(under) = 15 * 2840 * (580 - 50) = 22578000 \quad mm^3$

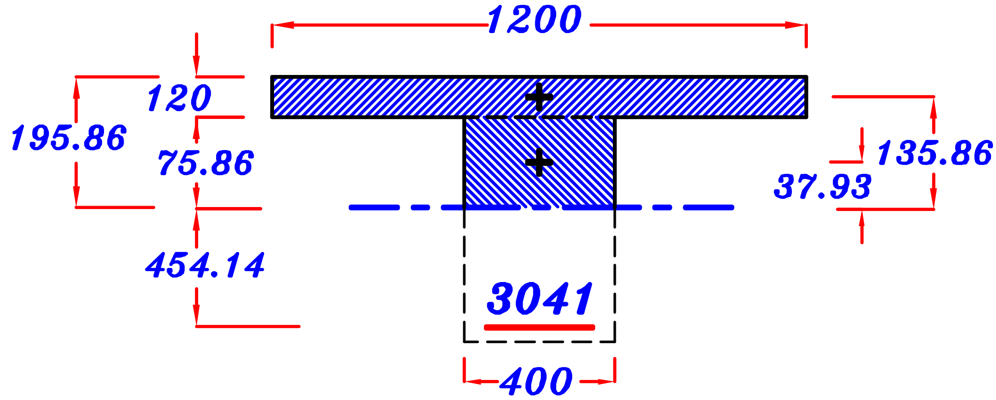
$\because S_{nv.}(under) > S_{nv.}(above) \quad \therefore Z > 120mm$



① Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

$$(1200)(120)(Z - 60) + (400)(Z - 120)\left(\frac{Z - 120}{2}\right) = (15)(3041)(650 - Z)$$

$$\boxed{Z = 195.86mm}$$



② $$I_{nv} = \frac{1200(120)^3}{12} + (1200)(120)(135.86)^2 + \frac{400(75.86)^3}{3} + (15)(3041)(454.14)^2 = 12296731390 \quad mm^4$$

③ $$M_{wc} = \frac{F_{cb} * I_{nv}}{Z}$$ --------- not as T_Sec.

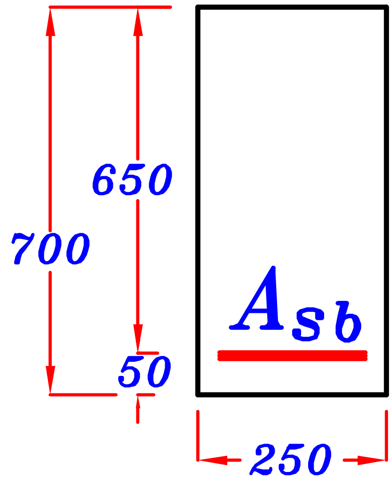
$$= \frac{9.5 * 12296731390}{195.86} = 596441071.1 \quad N.mm = 596.44 \quad kN.m$$

④ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{200}{15}\right) * 12296731390}{650 - 195.86} = 361026156 \; N.mm$$
$$= 361.02 \quad kN.m$$

⑤ $$\boxed{M_w = 361.02 \quad kN.m}$$

$$\boxed{Factor\ of\ Safty = \frac{M_{ult}}{M_w} = \frac{681.65}{361.02} = 1.89}$$

Example.



Data. $F_{cu} = 25$ N\mm^2

st. 360/520

Req. Calculate A_{sb} ($A_{s\ balanced}$)

To make the sec. is balanced Sec.

and then get M_b (M_{ult} For balanced sec)

Solution.

For Balanced Sec. $C = C_b$, $a = a_b = 0.8\ C_b$, $F_s = F_y$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25$ mm

② $a = a_b = 0.8\ C_b = 0.8 * 406.25 = 325$ mm

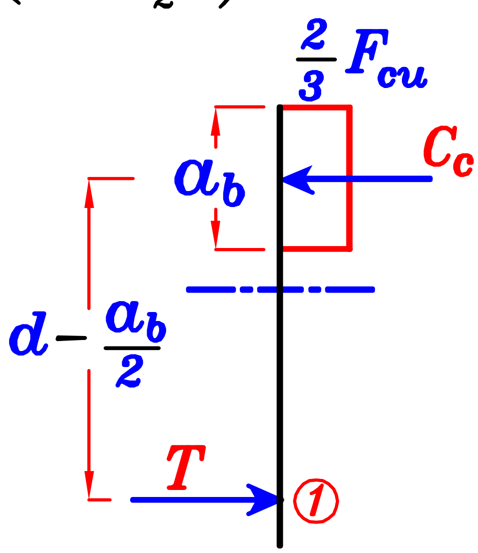
③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * (a_b * b) = A_{sb} * F_y$$

$$\frac{2}{3}(25)(325)(250) = A_{sb}(360) \quad \therefore \boxed{A_{sb} = 3761.5\ mm^2}$$

$$\therefore M_b = \frac{2}{3} F_{cu}\ a_b\ b\left(d - \frac{a_b}{2}\right) = \frac{2}{3}(25)(325)(250)\left(650 - \frac{325}{2}\right)$$

$$M_b = 660156250\ N.mm = 660.15\ kN.m$$



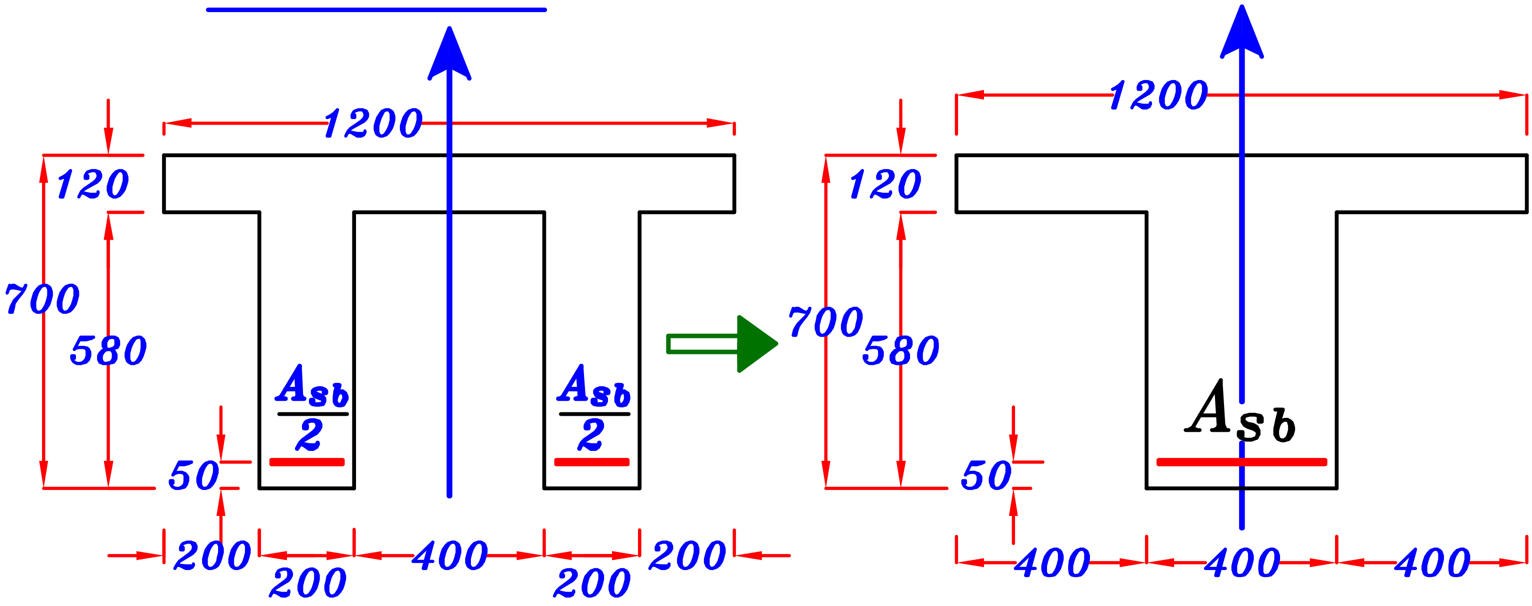
$$\text{or } M_b = A_{sb}\ F_y\left(d - \frac{a_b}{2}\right)$$

$$M_b = 3761.5\ (360)\left(650 - \frac{325}{2}\right)$$

$$M_b = 660156250\ N.mm = 660.15\ kN.m$$

$$\therefore \boxed{M_b = 660.15\ kN.m}$$

Example.



Data. $F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. Calculate A_{sb} To make the sec. is balanced Sec. and then get M_b

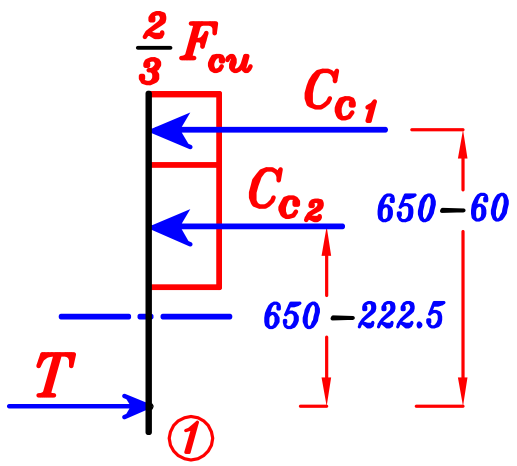
---

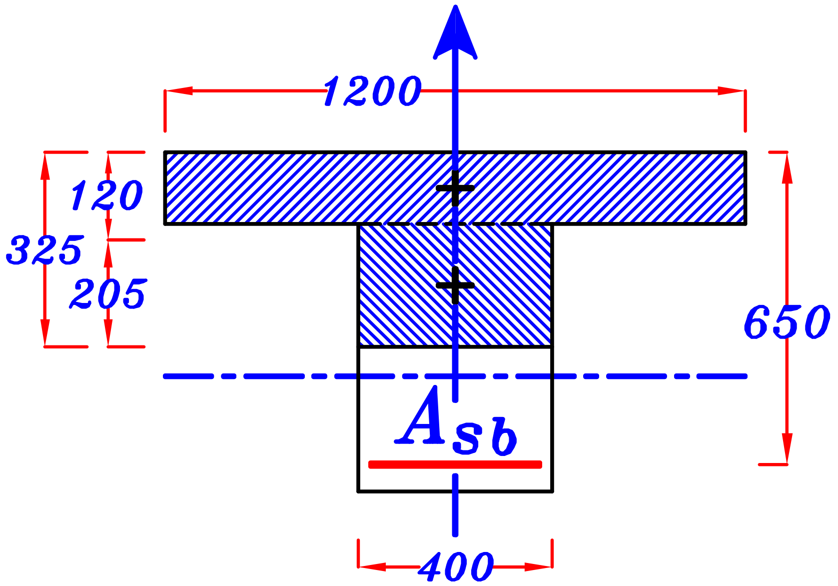
Solution.

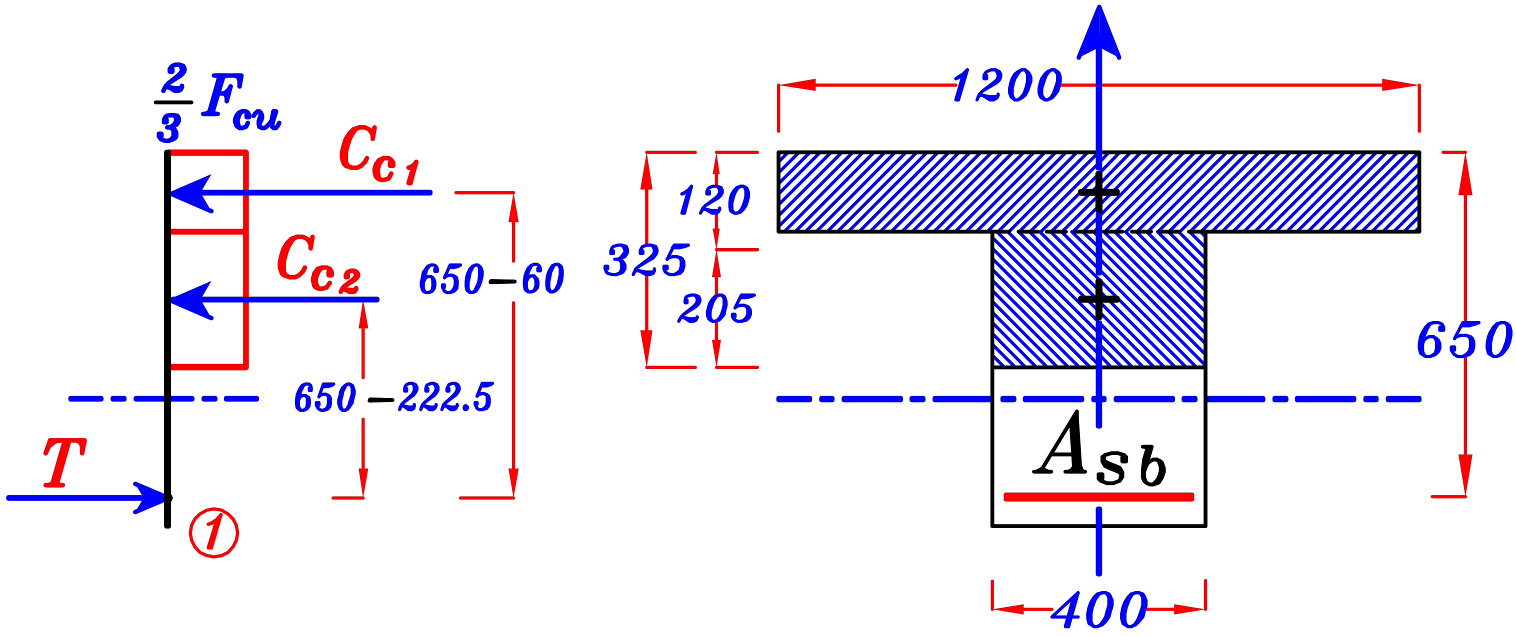
For Balanced Sec. $C = C_b$, $a = a_b = 0.8\ C_b$, $F_s = F_y$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25\ mm$

② $a = a_b = 0.8\ C_b = 0.8 * 406.25 = 325\ mm > t_s$







③ **From equilibrium eqn.** $C_{c1}+C_{c2}=T$

$$\frac{2}{3}F_{cu}*t_F*B+\frac{2}{3}F_{cu}*(a_b-t_s)*b=F_y*A_{sb}$$

$$\frac{2}{3}(25)(120)(1200)+\frac{2}{3}(25)(325-120)(400)=(360)\ A_{sb}$$

$$\therefore \boxed{A_{sb}=10463\ mm^2}$$

④ $\therefore M_{ult}=\frac{2}{3}F_{cu}\,t_s\,B\,(d-\frac{t_s}{2})+\frac{2}{3}F_{cu}\,(a_b-t_s)\,b\,(d-t_s-\frac{a-t_s}{2})$

$$M_{ult}=\frac{2}{3}(25)(120)(1200)\left(650-\frac{120}{2}\right)+\frac{2}{3}(25)\left(325-120\right)(400)\left(650-120-\frac{325-120}{2}\right)$$

$= 2000250000$ N.mm $= 2000.25$ kN.m

$$\therefore \boxed{M_b=M_{ult}=2000.25\ kN.m}$$

$(M_{U.L.})$

Introduction of Ultimate Limit Moment

Ultimate Limits Design Method هو العزم الذى تم عليه تصميم القطاع بطريقه

و للتصميم بهذه الطريقه يجب الاخذ فى الاعتبار قيم **Factor Of Safty**

Factors Of Safty For Limit State Design.

* F.O.S. For Loads.

F.O.S. For Dead Load. $= 1.4$
F.O.S. For Live Load. $= 1.6$
} To increase the Load.

F.O.S. For Dead Load. $= 0.9$
F.O.S. For Live Load. $=$ zero
} To decrease the Load.

Load (To Increase) $= 1.4$ D.L. $+ 1.6$ L.L.

$= 1.5$ (D.L.+ L.L.) IF L.L. $\geqslant 0.75$ D.L.

Load (To Decrease) $= 0.9$ D.L. $+ 0.0$ L.L.

* F.O.S. For Materials.

1- Case of Axial and eccentric load. (M, N)

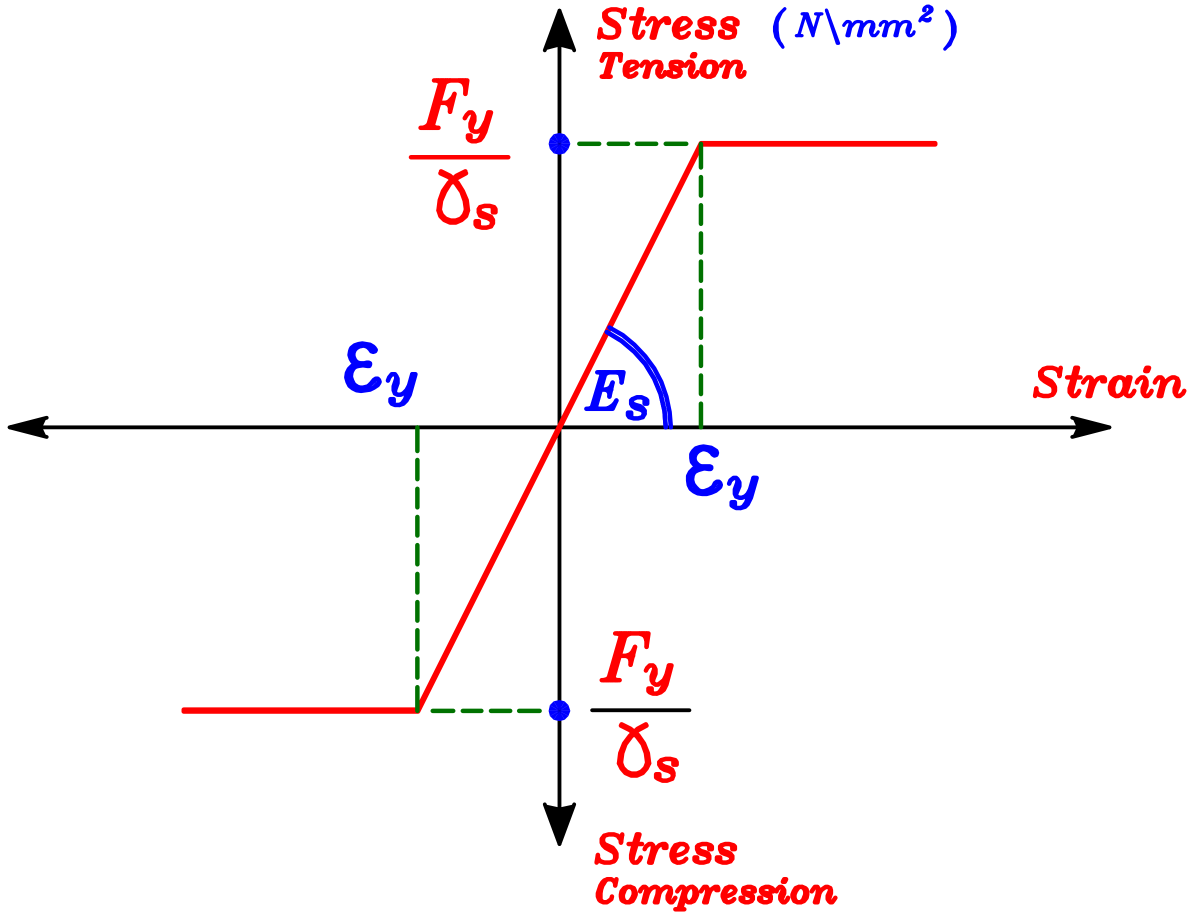
$$\boxed{\begin{array}{l} \gamma_c \text{ (Concrete)} = 1.5\left[\left(\frac{7}{6}\right) - \frac{(e \backslash t)}{3}\right] \geqslant 1.5 \\ \gamma_s \text{ (Steel)} = 1.15\left[\left(\frac{7}{6}\right) - \frac{(e \backslash t)}{3}\right] \geqslant 1.15 \end{array}}$$

2- Case of Flexure only. (M) only

$$\boxed{\gamma_c = 1.5 \quad , \quad \gamma_s = 1.15}$$ حفظ

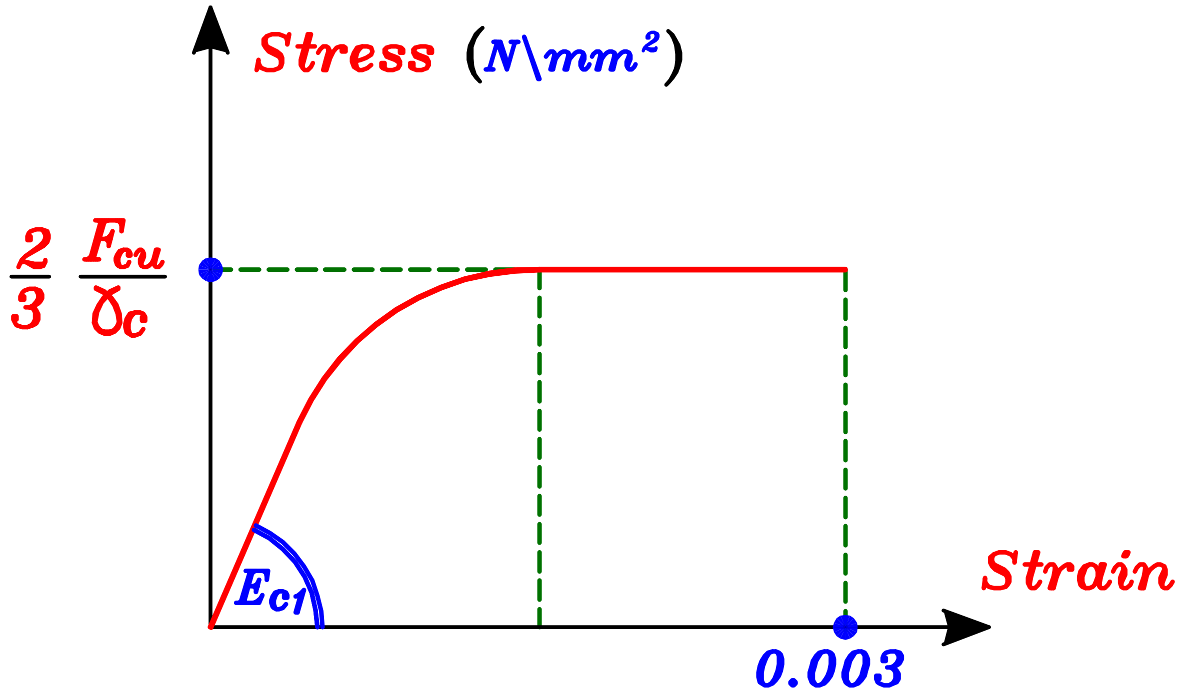
$\therefore$ Allowable stress For concrete. $= \left(\frac{F_{cu}}{\gamma_c}\right)$

Allowable stress For steel. $= \left(\frac{F_y}{\gamma_s}\right)$



Idealized Stress-Strain Curve For Steel.

المنحنى الاعتبارى للاجهاد و الانفعال للحديد .

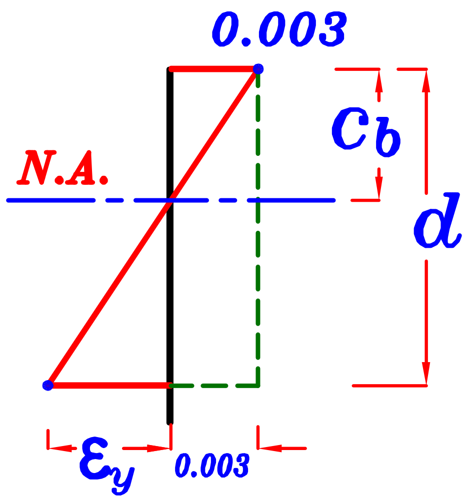


Idealized Stress-Strain Curve For Concrete.

المنحنى الاعتبارى للاجهاد و الانفعال للخرسانه

Properties of Under Reinforced Section.

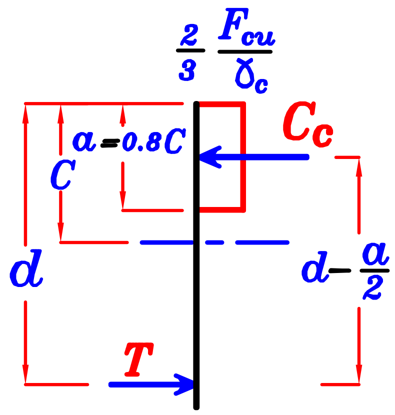
$$\because C_b = \frac{600}{600 + (F_y \backslash \gamma_s)} * d$$



For under Reinforced section $C \leqslant C_b$

① $C \leqslant C_{max.}$ where: $\boxed{C_{max} = \frac{2}{3} C_b}$

$$\therefore \boxed{C_{max} = \frac{2}{3}\left[\frac{600}{600 + (F_y \backslash \gamma_s)} * d\right]}$$



IF $C > C_{max.}$ $\longrightarrow$ over reinforced sec. نعتبر كأن القطاع
و هذا لا ينفع فى التصميم

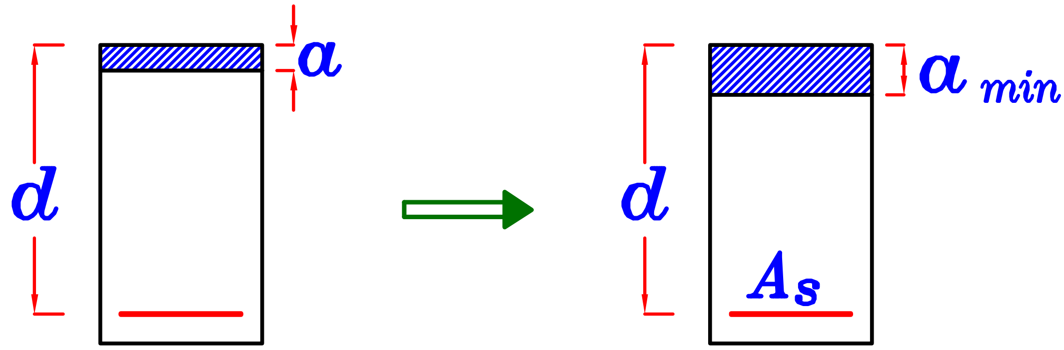
② $a \leqslant a_{max.}$ $\boxed{a_{max.} = 0.8\ C_{max.}}$

$$\therefore \boxed{a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)} * d\right]}$$

IF $a > a_{max.}$ $\longrightarrow$ over reinforced sec. نعتبر كأن القطاع
و هذا لا ينفع فى التصميم

③ $a \geqslant a_{min}$ $\boxed{a_{min} = 0.1\,d}$

عند التصميم يجب عمل **check** على a بحيث لا تكون أقل من a_{min}

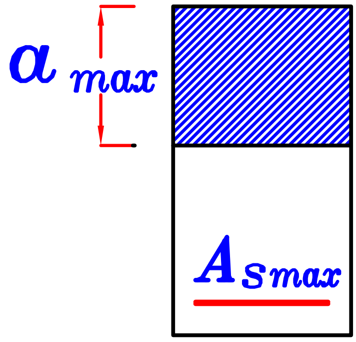


IF $a < a_{min}$ $\xrightarrow{Take}$ $a = a_{min}$

عن طريق زياده كميه الحديد

---

④ $A_s \leqslant A_{s_{max.}}$



IF $A_s = A_{s_{max.}} \longrightarrow a = a_{max.}$

IF $A_s > A_{s_{max.}} \longrightarrow a > a_{max.} \longrightarrow$ *over reinforced sec.*

و هذا لا ينفع فى التصميم

To Calculate $A_{s_{max.}}$ $\boxed{A_{s_{max.}} = \mu_{max.}\, b\, d}$

Where:

$$\mu = \frac{A_s}{bd} = \frac{\text{مساحة الحديد الرئيسى}}{\text{مساحه الخرسانه}}$$

$$\mu_{max.} = \frac{A_{s_{max.}}}{bd}$$ $\longrightarrow$ Code *Page* (4–7) *Table* (1–4)

جدول (٤-١) معامل الحد الأقصى لمقاومة العزوم R_{max} ونسبة صلب التسليح القصوى μ_{max}
ونسبة العمق الأقصى لمحور الخمول إلى العمق الفعال c_{max}/d للقطاعات المسلحة جهة الشد فقط

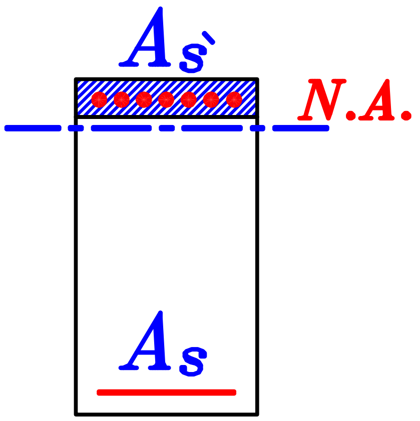
رتبة الصلب*	c_{max}/d	μ_{max}	R_{max}
240/350	0.50	$8.56\text{x}10^{-4}\ f_{cu}$	0.214
280/450	0.48	$7.00\text{x}10^{-4}\ f_{cu}$	0.208
360/520	0.44	$5.00\text{x}10^{-4}\ f_{cu}$	0.194
400/600	0.42	$4.31\text{x}10^{-4}\ f_{cu}$	0.187
450/520**	0.40	$3.65\text{x}10^{-4}\ f_{cu}$	0.180

* طبقاً للجدول (٢-١) وحيث f_{cu} بوحدات ن/مم٢ .

** خاصة لصلب الشبك مع استيفاء ما جاء بالبند (٤-٢-١-١-٣) .

⑤ $A_{s`} \leqslant A_{s`max.}$ IF we are using $A_{s`}$

where $$\boxed{A_{s`max.} = 0.4\ A_s}$$



اذا كانت $A_{s`} > A_{s`max.}$ سيكون ال N.A. قريب جدا من ال Compression side
و فى نفس الوقت سيكون صب خرسانه الكمره
صعب جدا (ممكن أن تعشش الخرسانه).

ممكن استخدام هذه المعادله لكن يجب اثباتها أولا .

$$\boxed{A_{s`max.} = 0.4\ A_s = \frac{2}{3}\ \mu_{max.}\ b\ d}$$

إثبات

$$A_{s`max.} = 0.4\ A_s = 0.4\left(A_{s_{max.}} + A_{s`max.}\right)$$

$$\therefore\ A_{s`max.} = 0.4\left(\mu_{max.}\ b\ d + A_{s`max.}\right)$$

$$\therefore\ A_{s`max.} = 0.4\ \mu_{max.}\ b\ d + 0.4\ A_{s`max.}$$

$$\therefore\ 0.6\ A_{s`max.} = 0.4\ \mu_{max.}\ b\ d$$

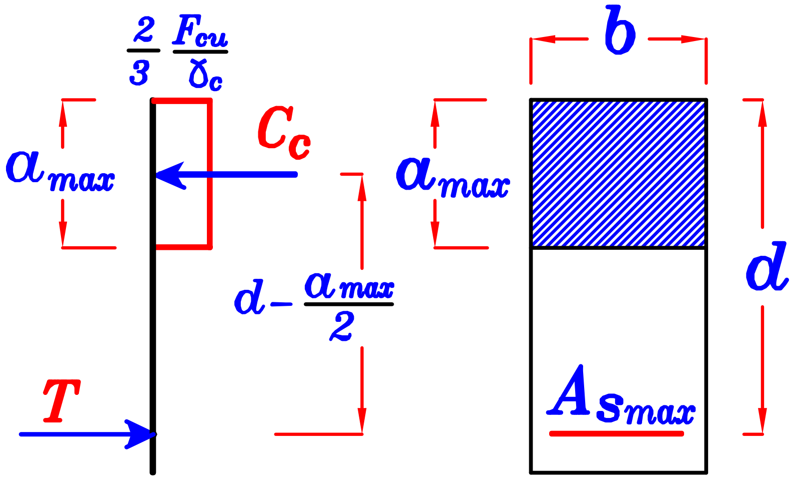
$$\therefore\ \boxed{A_{s`max.} = \frac{2}{3}\ \mu_{max.}\ b\ d}$$

⑥ $M_{U.L._{max}}$

هو أكبر عزم يتحمله قطاع ذو d معلومه و يظل **under reinforced section**

و لحساب قيمه $M_{U.L._{max}}$ نأخذ $A_s = A_{s_{max}}$ و بالتالى تكون $a = a_{max}$

Without $A_{s'}$



Code Page (4–7)

Table (1–4)

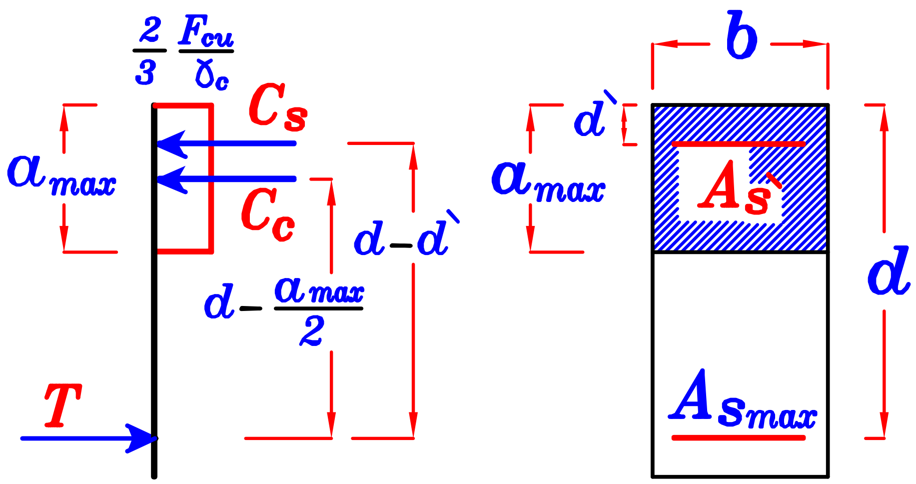
$$\boxed{\begin{aligned} M_{U.L._{max}} &= \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a_{max}\, b \left(d - \frac{a_{max}}{2}\right) \\ M_{U.L._{max}} &= R_{max.}\frac{F_{cu}}{\gamma_c}\, b\, d^2 \end{aligned}}$$

With $A_{s'}$

$\boxed{A_{s'} = A_{s'_{max}}}$ نأخذ

$\boxed{A_{s'_{max.}} = \frac{2}{3}\,\mu_{max.}\, b\, d}$

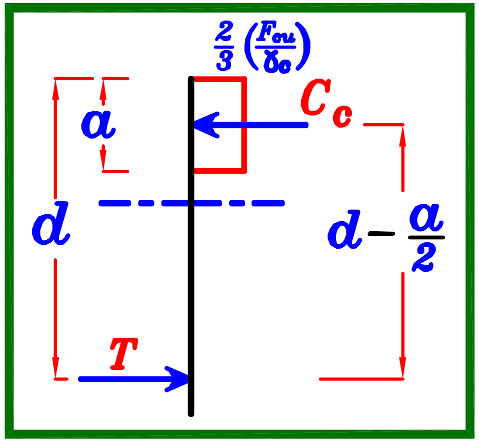
مع اثباتها أولا



$$\boxed{\begin{aligned} M_{U.L._{max}} &= \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a_{max}\, b \left(d - \frac{a_{max}}{2}\right) + A_{s'_{max}}\frac{F_y}{\gamma_s}(d - d') \\ M_{U.L._{max}} &= R_{max.}\frac{F_{cu}}{\gamma_c}\, b\, d^2 + A_{s'_{max}}\frac{F_y}{\gamma_s}(d - d') \end{aligned}}$$

Calculation of $M_{U.L.}$ (With Ten. Steel Only)

Calculate $a_{max} = 0.8\left(\frac{2}{3}\right) C_b = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = F_s * A_s$

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s \longrightarrow$ Get a

IF $\boxed{a}$

IF $a \leqslant 0.1\,d$

take $a = 0.1\,d$

$\boxed{\text{يجب أخذ العزم عند الخرسانه}}$

$\therefore M_{U.L.} = A_s \frac{F_y}{\gamma_s}\left(d - \frac{a}{2}\right)$

$\therefore M_{U.L.} = A_s \frac{F_y}{\gamma_s}\left(d - \frac{0.1d}{2}\right)$

$\therefore M_{U.L.} = A_s F_y\, d\, \frac{1}{1.15}\left(1 - \frac{0.1}{2}\right)$

$\therefore \boxed{M_{U.L.} = 0.826\, A_s\, F_y\, d}$

$0.1\,d < a < a_{max.}$

Right Assumption $F_s = \frac{F_y}{\gamma_s}$

$\boxed{\text{يؤخذ العزم عند الحديد أو الخرسانه}}$

$\therefore$
$$\boxed{\begin{gathered} M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right) \\ M_{U.L.} = A_s * \frac{F_y}{\gamma_s}\left(d - \frac{a}{2}\right) \end{gathered}}$$

IF $a > a_{max.}$

Wrong Assumption $F_s \neq \frac{F_y}{\gamma_s}$

Take $a = a_{max.}$

$\boxed{\text{يجب أخذ العزم عند الحديد}}$

$\therefore$
$$\boxed{\begin{gathered} M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a_{max.}\, b\left(d - \frac{a_{max.}}{2}\right) \\ M_{U.L.} = R_{max.}\frac{F_{cu}}{\gamma_c}\, b\, d^2 \end{gathered}}$$

Example.

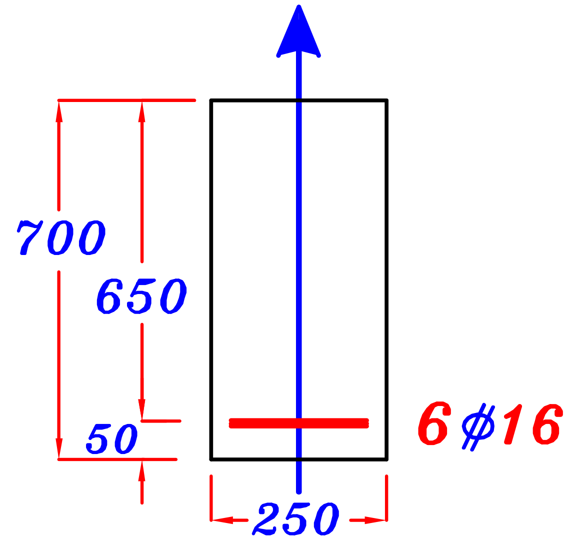
Data.

$F_{cu} = 25 \; N \backslash mm^2$

st. 360/520

Req.

Calculate $M_{U.L.}$



Solution.

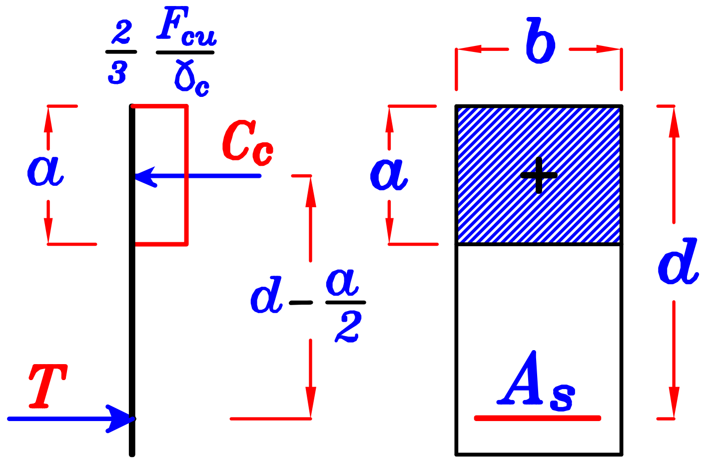
$$A_s = 6\phi16 = 6\left[\frac{\pi * 16^2}{4}\right] = 1206 \; mm^2$$

$$a_{min} = 0.1 \; d = 0.1 * 650 = 65 \; mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35 \; d = 0.35 * 650 = 227.5 \; mm$$

$$C_c = Stress * Area = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * a * b$$

$$T = Stress * Area = F_s * A_s$$



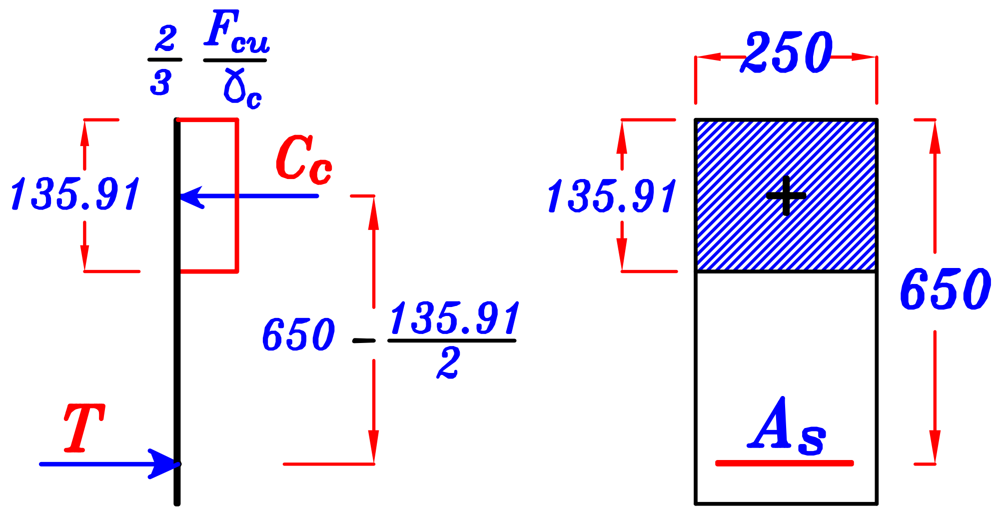
From equilibrium eqn. $\frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * a * b = A_s * F_s$ ----- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\therefore \; \frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) = (1206)\left(\frac{360}{1.15}\right) \longrightarrow \boxed{a = 135.91 \, mm}$$

$\therefore\ 0.1\,d < a < a_{max.}$ Right assumption $F_s = \frac{F_y}{\delta_s}$



By taking the moment about the steel.

$$M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\delta_c}\,a\,b\left(d - \frac{a}{2}\right)$$

$$M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(135.91)(250)\left(650 - \frac{135.91}{2}\right)$$

$$= 219738155.4\ N.mm = 219.73\ kN.m$$

OR take the moment about the concrete.

$$M_{U.L.} = A_s\,\frac{F_y}{\delta_s}\left(d - \frac{a}{2}\right)$$

$$M_{U.L.} = 1206\left(\frac{360}{1.15}\right)\left(650 - \frac{135.91}{2}\right)$$

$$= 219739701.9\ N.mm = 219.74\ kN.m$$

$$\boxed{M_{U.L.} = 219.73\ kN.m}$$

الفرق فى قيمتى العزم ناتج فقط عن التقريب
لكن كلا الاجابتين صحيح .

Example.

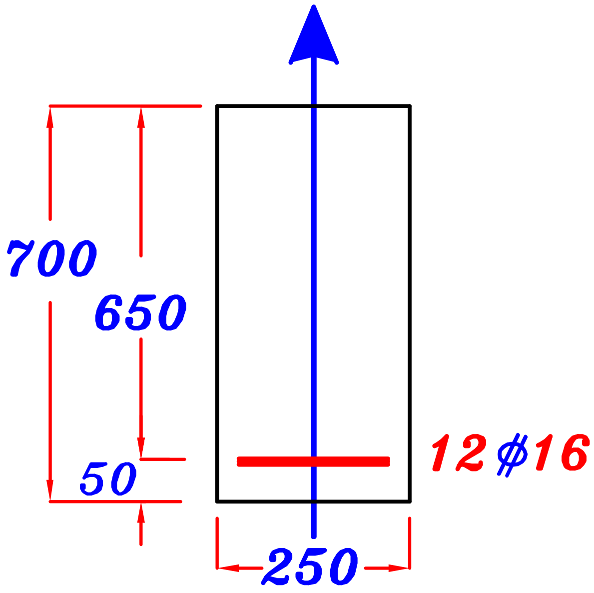
Data.

$F_{cu} = 25\ N\backslash mm^2$

st. 360/520

Req.

Calculate $M_{U.L.}$



Solution.

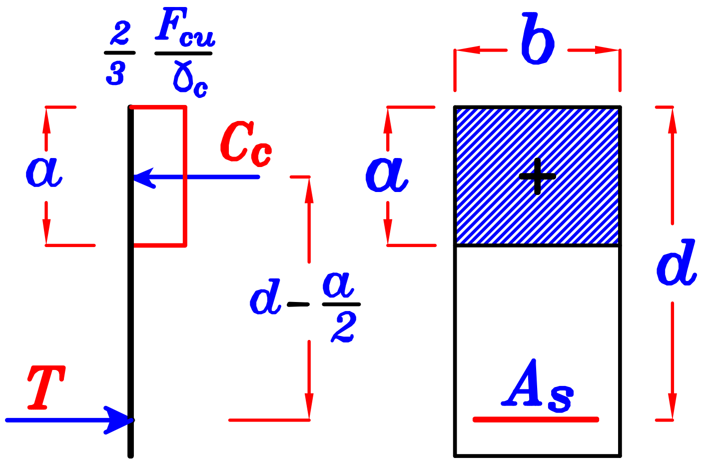
$$A_s = 12\#16 = 12\left[\frac{\pi * 16^2}{4}\right] = 2412\ mm^2$$

$$a_{min} = 0.1\ d = 0.1 * 650 = 65\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 650 = 227.5\ mm$$

$$C_c = Stress * Area = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b$$

$$T = Stress * Area = F_s * A_s$$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = F_s * A_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) = \left(\frac{360}{1.15}\right)(2412) \longrightarrow \boxed{a = 271.82 mm}$$

$\therefore \quad a > a_{max.} \longrightarrow$ **Take** $a = a_{max.}$

$$M_{U.L.} = \frac{2}{3} \frac{F_{cu}}{\gamma_c} a_{max.} b \left(d - \frac{a_{max.}}{2}\right)$$

$$M_{U.L.} = \frac{2}{3} \left(\frac{25}{1.5}\right) (227.5)(250)\left(650 - \frac{227.5}{2}\right)$$

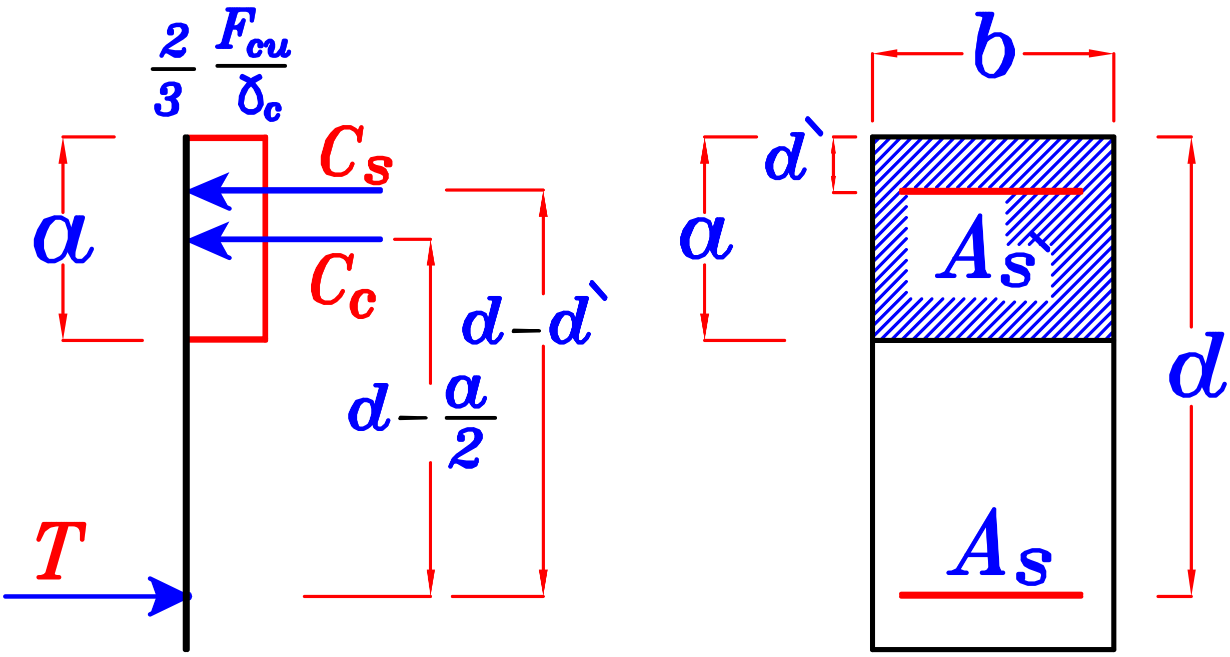
$$= 338880208.3 \text{ N.mm} = 338.88 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 338.88 \text{ kN.m}}$$

عند حساب $M_{U.L.}$ وكان هناك حديد جهه الضغط $(A_{s`})$

نعمل حل تقريبى للتسهيل بأن نعتبر $F_{s`} = \frac{F_y}{\gamma_s}$

و لحساب الـ $M_{U.L.}$ مع وجود $(A_{s`})$ بدقه سنذكرها فى أخر الملف *Page No. 103*



$$C_c = Stress * Area = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * (a\ b)$$

$$C_s = Stress * Area = \frac{F_y}{\gamma_s} * A_{s`}$$

By taking the moment about the steel.

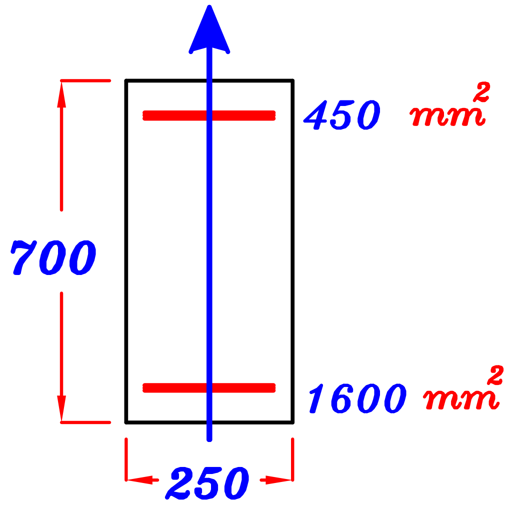
$$\boxed{M_{ult} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\, a\, b\, \left(d - \frac{a}{2}\right) + \frac{F_y}{\gamma_s} * A_{s`}\,(d - d`)}$$

Example.

Data.

$F_{cu} = 25 \ N \backslash mm^2$ st. 360/520

Req. Calculate $M_{U.L.}$



Solution. $\because \dfrac{A_{s`}}{A_s} = \dfrac{450}{1600} = 0.28 > 0.2 \quad \therefore$ Use $A_{s`}$

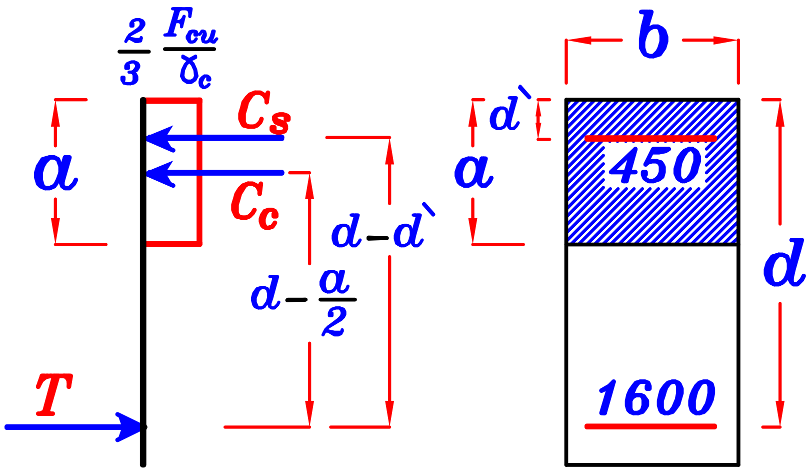
$$a_{min} = 0.1\ d = 0.1 * 650 = 65 \ mm$$

$$a_{max} = 0.8 \left(\frac{2}{3}\right) \left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 650 = 227.5 \ mm$$

$$C_c = Stress * Area = \frac{2}{3} \frac{F_{cu}}{\gamma_c} * a * b$$

$$C_s = Stress * Area = \frac{F_y}{\gamma_s} * A_{s`}$$

$$T = Stress * Area = F_s * A_s$$



From equilibrium eqn. $\dfrac{2}{3} \dfrac{F_{cu}}{\gamma_c} * a * b + \dfrac{F_y}{\gamma_s} * A_{s`} = F_s * A_s$

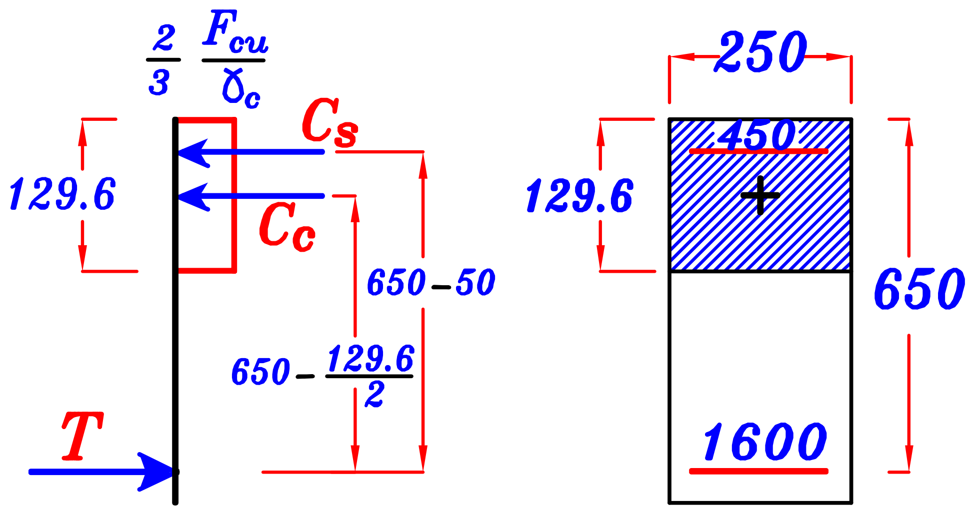
assume $F_s = \dfrac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\frac{2}{3} \frac{F_{cu}}{\gamma_c} * a * b + \frac{F_y}{\gamma_s} * A_{s`} = \frac{F_y}{\gamma_s} * A_s$$

$$\frac{2}{3} \left(\frac{25}{1.5}\right) (a) \ (250) + \left(\frac{360}{1.15}\right) (450) = \left(\frac{360}{1.15}\right) (1600)$$

$$\boxed{a = 129.6 \ mm}$$

$\therefore\ 0.1\,d < a < a_{max.}$ **Right assumption** $F_s = \frac{F_y}{\gamma_s}$



By taking the moment about the steel.

$$M_{U.L.} = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right) + \frac{F_y}{\gamma_s} * A_{s`}\,(d - d`)$$

$$M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(129.6)(250)\left(650 - \frac{129.6}{2}\right) + \left(\frac{360}{1.15}\right)(450)\,(650 - 50)$$

$$M_{U.L.} = 295193739\ \text{N.mm} = 295.19\ \text{kN.m}$$

$$\boxed{M_{U.L.} = 295.19\ \text{kN.m}}$$

Example.

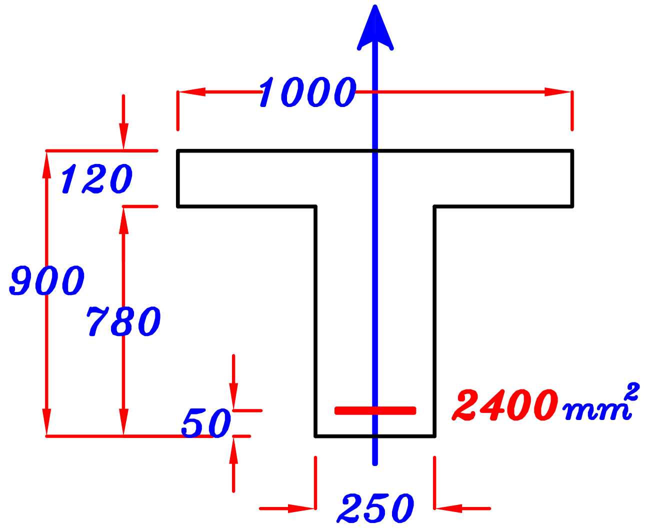
Data.

$F_{cu} = 25\ N\backslash mm^2$

st. 360/520

Req.

Calculate $M_{U.L.}$



Solution.

$a_{min} = 0.1\ d = 0.1 * 850 = 85\ mm$

$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\ d = 0.35 * 850 = 297.5\ mm$

assume $a \leqslant t_s$ $\quad a < 120\ mm$

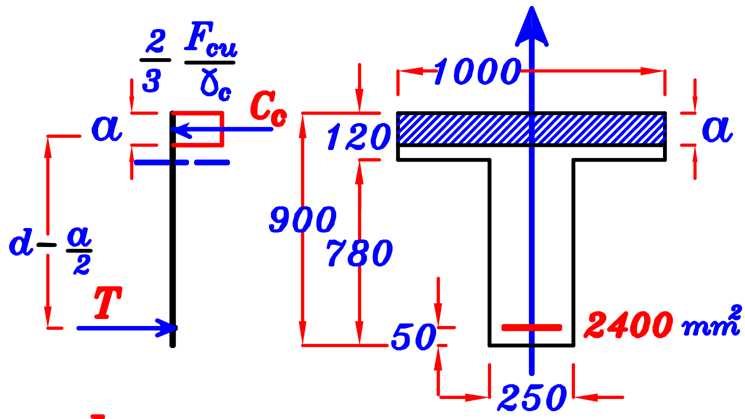
From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = \frac{F_y}{\gamma_s} * A_s$

$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(1000) = \left(\frac{360}{1.15}\right)(2400)$

$\longrightarrow a = 67.6\ mm < t_s \quad \therefore$ o.k.



, $a < 0.1\,d \quad \therefore$ take $a = 0.1\,d = 85\ mm$

$M_{U.L.} = \frac{F_y}{\gamma_s} A_s\left(d - \frac{a}{2}\right) = \left(\frac{360}{1.15}\right) 2400 \left(850 - \frac{85}{2}\right)$

$= 606678260.9\ N.mm = 606.67\ kN.m$

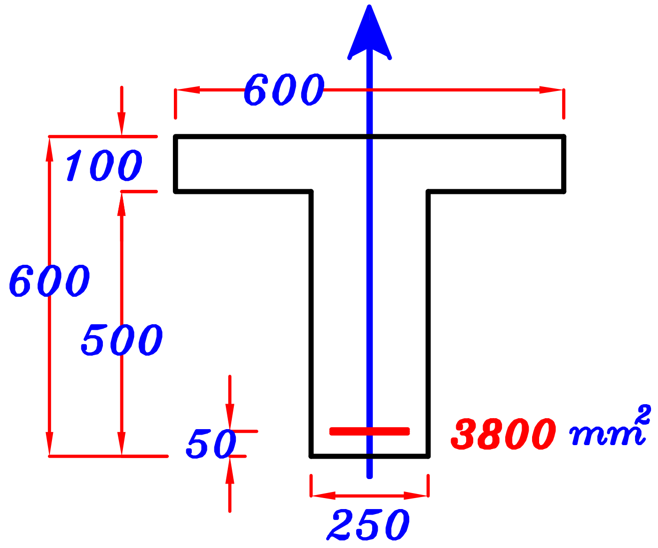
$$\boxed{M_{U.L.} = 606.67\ kN.m}$$

Example.

Data.

$F_{cu} = 25 \ N\backslash mm^2$

$st. \ 360/520$



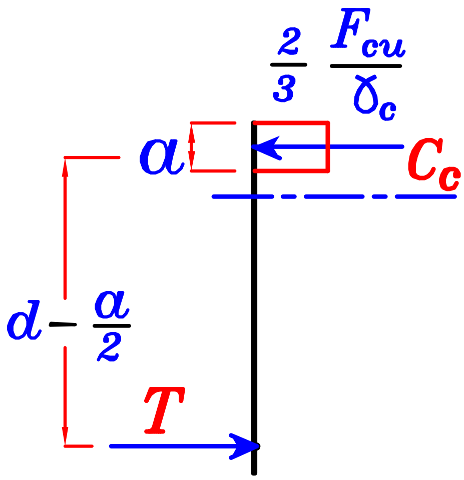
Req. Calculate $M_{U.L.}$

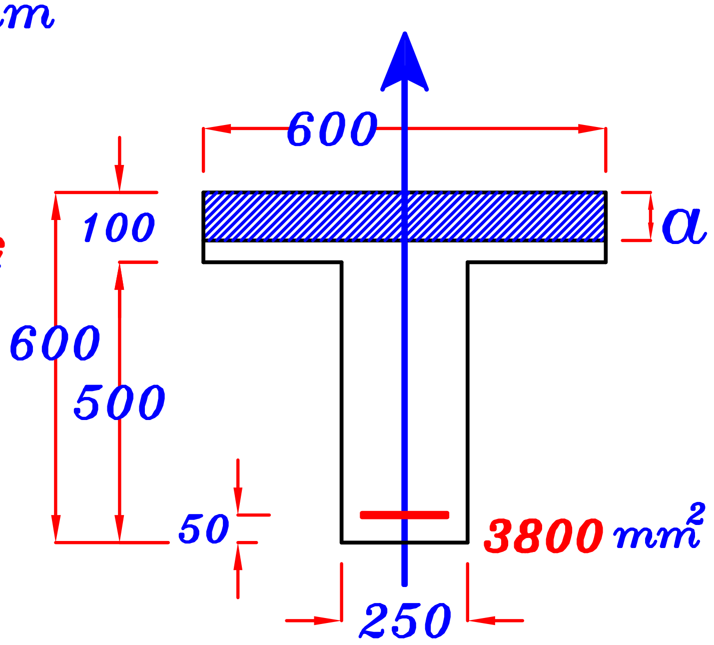
Solution.

$$a_{min} = 0.1 \ d = 0.1 * 550 = 55 \ mm$$

$$a_{max} = 0.8 \left(\frac{2}{3}\right) \left[\frac{6000}{6000 + (F_y \backslash \gamma_s)}\right] * d = 0.35 \ d = 0.35 * 550 = 192.5 \ mm$$

assume $a \leqslant t_s \qquad a < 100 \ mm$





From equilibrium eqn. $C_c = T$

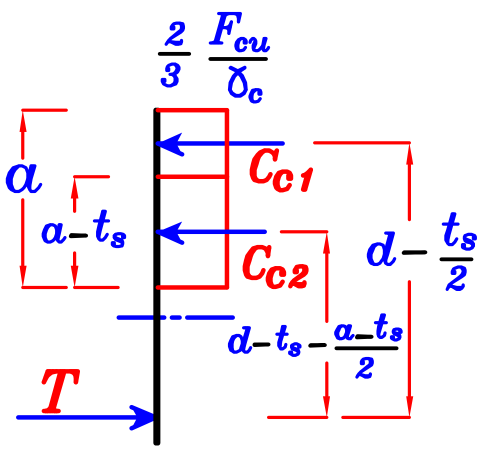
$$\frac{2}{3} \frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s \ \text{----} \ a \ , F_s$$

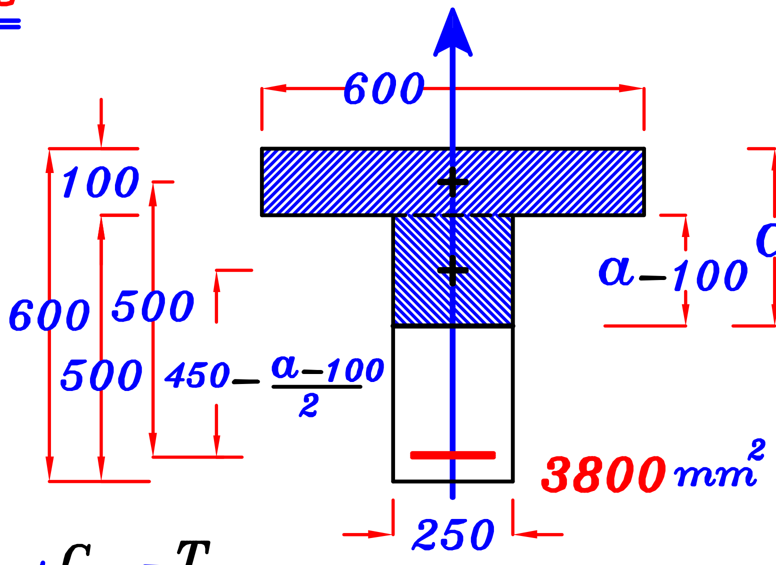
Assume $F_s = \dfrac{F_y}{\gamma_s} \longrightarrow$ (under reinforced Sec.)

$$\frac{2}{3} \left(\frac{25}{1.5}\right) (a) (600) = \left(\frac{360}{1.15}\right) (3800) \longrightarrow a = 178.4 \ mm > t_s$$

$a > t_s$ wrong assumption $\therefore$ Take $a > t_s$

$\therefore$ **Take** $a > t_s$





From equilibrium eqn. $C_{c1} + C_{c2} = T$

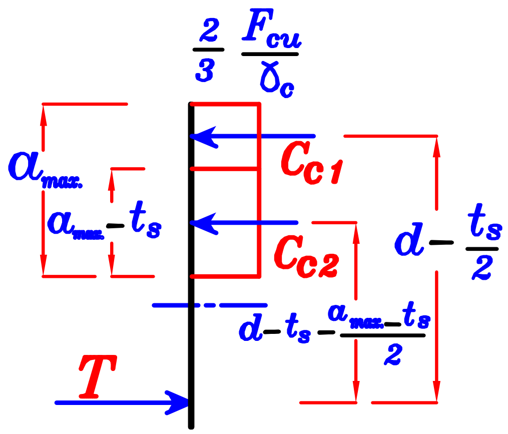
$$\frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * t_s * B + \frac{2}{3}\,\frac{F_{cu}}{\gamma_c} *(a - t_s) * b = F_s * A_s$$

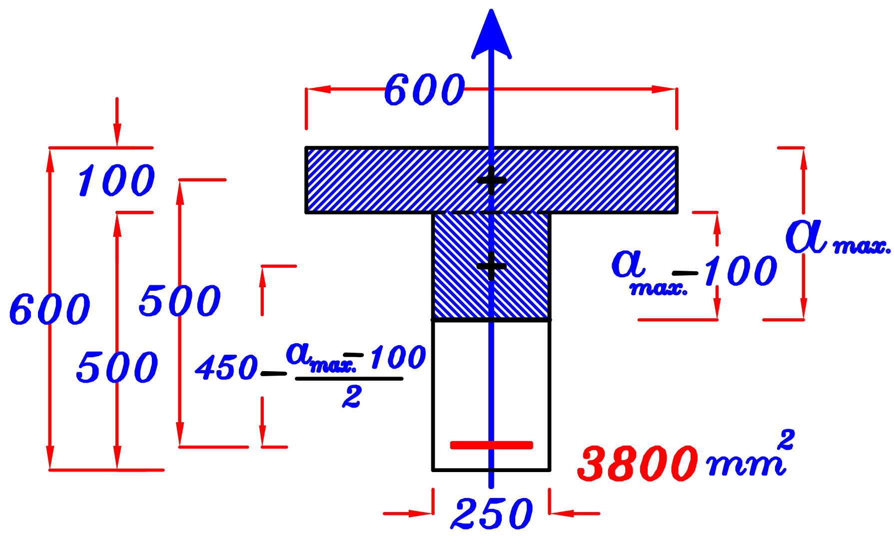
Assume $F_s = \dfrac{F_y}{\gamma_s} \longrightarrow$ (*under reinforced Sec.*)

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(100)(600) + \frac{2}{3}\left(\frac{25}{1.5}\right)(a - 100)(250) = \left(\frac{360}{1.15}\right)(3800)$$

$\longrightarrow a = 288.24 \text{ mm}$

$\therefore a > a_{max.} \longrightarrow$ **Take** $a = a_{max.} = 192.5 \text{ mm}$





$$M_{U.L.} = \left(\frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * t_s * B\right)\left(d - \frac{t_s}{2}\right) + \left(\frac{2}{3}\,\frac{F_{cu}}{\gamma_c} *(a_{max.} - t_s) * b\right)\left(d - t_s - \frac{a_{max.} - t_s}{2}\right)$$

$$M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(100)(600)\left(550 - \frac{100}{2}\right) + \frac{2}{3}\left(\frac{25}{1.5}\right)(192.5 - 100)(250)\left(550 - 100 - \frac{192.5 - 100}{2}\right)$$

$$M_{U.L.} = 437074652.8 \text{ N.mm} = 437.07 \text{ kN.m}$$

$$\therefore \boxed{M_{U.L.} = 437.07 \text{ kN.m}}$$

Example.

For the section it is required to calculate :

a – The Cracking Moment. $(M_{cr.})$

b – The Working Moment. (M_w)

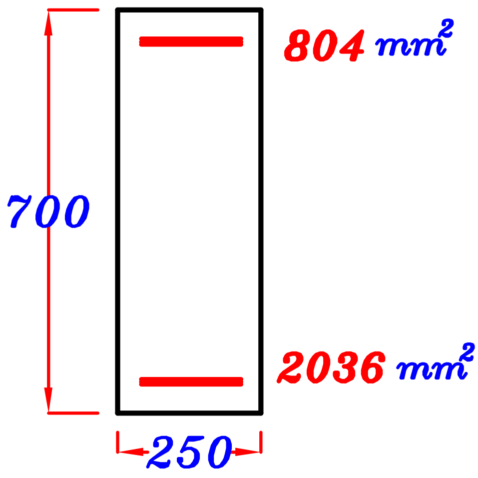
c – The Failure Moment. (M_{ult})

d – The Ultimate Limit Moment. $(M_{U.L.})$

e – The Factor Of Safty For Loads.

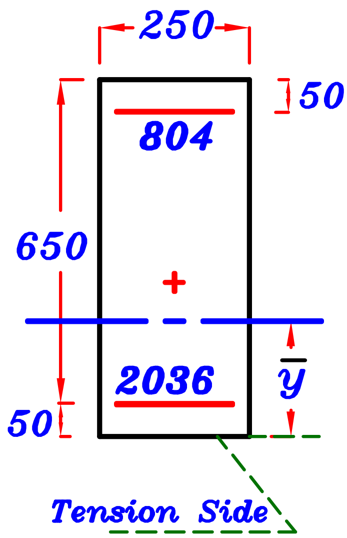
f – The Factor Of Safty For Material.

g – The Global Factor Of Safty.



Data : $F_{cu} = 25 \ kN\backslash m^2$, st. 360/520

a– M_{cr}



① $n = \dfrac{E_s}{E_c} = \dfrac{2 * 10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$

② $A_v = b * t + (n-1)A_s + (n-1)A_s`$

$A_v = 250 * 700 + (10-1)(2036) + (10-1)(804) = 200560 \ mm^2$

③ $\bar{y}_t = \dfrac{250 * 700 * 350 + (10-1)(2036)(50) + (10-1)(804)(650)}{200560} = 333.4 \ mm$

④ $I_{gross} = \dfrac{250 * 700^3}{12} + 250 * 700(350 - 333.4)^2 + (10-1)(2036)(333.4 - 50)^2$

$+ (10-1)(804)(650 - 333.4)^2 = 9391063167 \ mm^4$

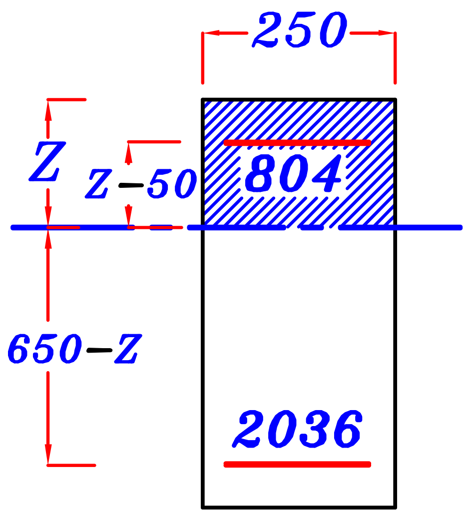
⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0 \ N\backslash mm^2$

⑥ $M_{cr} = \dfrac{F_{ctr} * I_g}{\bar{y}_t} = \dfrac{3.0 * 9391063167}{333.4} = 84502668 \ mm.N$

$= 84.5 \ kN.m$

$$\boxed{M_{cr} = 84.5 \ kN.m}$$

b – M_w



Allowable stresses

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$

① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_{s'}\,(Z - d') = n\,A_s\,(d - Z)$$

$$250\,(Z)\left(\frac{Z}{2}\right) + (14)(804)\,(Z - 50) = (15)\,(2036)\,(650 - Z)$$

$$\boxed{Z = 270.1 \ mm}$$

③ Get $I_{nv} = \dfrac{b\,Z^3}{3} + (n-1)\,A_{s'}\,(Z - d')^2 + n\,A_s\,(d - Z)^2$

$$I_{nv} = \frac{250\,(270.1)^3}{3} + (14)\,(804)\,(270.1 - 50)^2 + (15)\,(2036)\,(650 - 270.1)^2$$

$$= 6595014217 \ mm^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 6595014217}{270.1} = 231960885 \ N.mm$

$= 231.9 \ kN.mm$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 6595014217}{650 - 270.1} = 231464919 \ N.mm$

$= 231.46 \ kN.m$

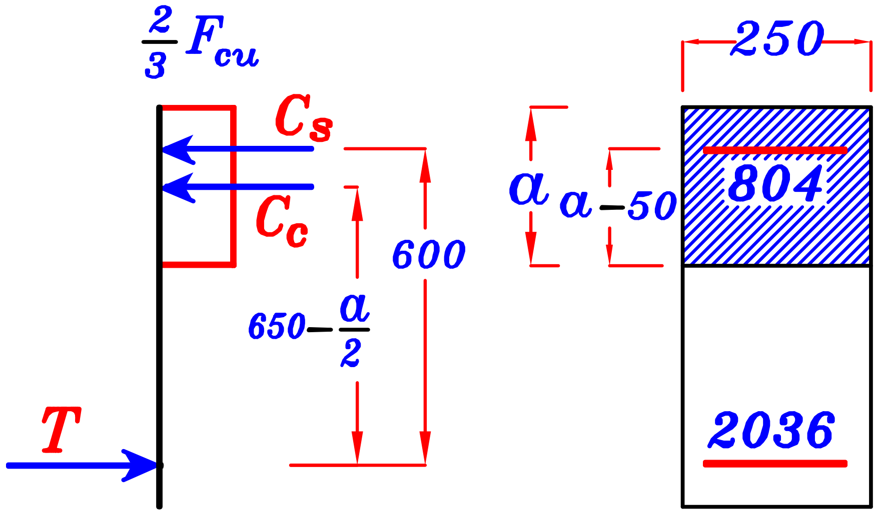
⑥ $\boxed{M_w = 231.46 \ kN.m}$

c – Mult.

ملحوظه : للتسهيل نأخذ ال stress على حديد الضغط يساوى F_y

$d = 650\ mm$

$d' = 50\ mm$



① $C_b = \frac{600}{600+F_y} * d = \frac{600}{600+360} * 650 = 406.25\ mm$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3}F_{cu} * (a*b) + F_y * A_{s'} = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(250) + (360)(804) = (360)(2036)$$

$\longrightarrow a = 106.4\ mm \longrightarrow C = 1.25\,a = 1.25 * 106.4 = 133.0\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3}F_{cu}\,a\,b\left(d-\frac{a}{2}\right) + F_y\,A_{s'}(d-d')$$

$$= \frac{2}{3}(25)(106.4)(250)\left(650-\frac{106.4}{2}\right) + (360)(804)(650-50)$$

$= 438245333\ N.mm = 438.2\ kN.m$

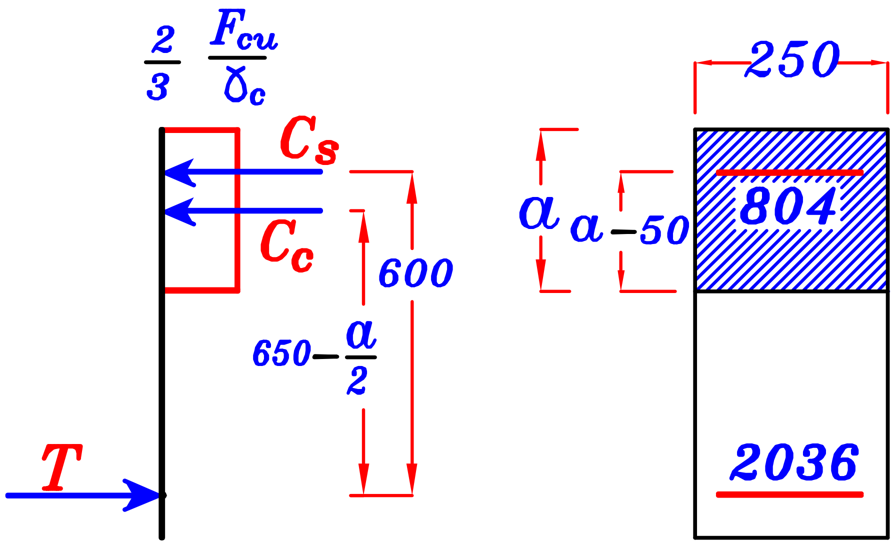
$$\boxed{M_{ult} = 438.2\ kN.m}$$

$d - \underline{\underline{M_{U.L.}}}$

ملحوظه : للتسهيل نأخذ ال stress على حديد الضغط يساوى $\frac{F_y}{\gamma_s}$

$d = 650\ mm$

$d` = 50\ mm$



$$a_{min} = 0.1\, d = 0.1 * 650 = 65\ mm$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\, d = 0.35 * 650 = 227.5\ mm$$

From equilibrium eq$\underline{n}$. $C_c + C_s = T$

$$\frac{2}{3}\,\frac{F_{cu}}{\gamma_c} * (a * b) + \frac{F_y}{\gamma_s} * A_{s`} = F_s * A_s$$

Assume $F_s = \frac{F_y}{\gamma_s} \longrightarrow$ (under reinforced)

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) + \left(\frac{360}{1.15}\right)(804) = \left(\frac{360}{1.15}\right)(2036)$$

$\longrightarrow a = 138.8\ mm \qquad \therefore\ 0.1\, d < a < a_{max}$

Right assumption $F_s = \frac{F_y}{\gamma_s}$

$$\therefore\ M_{U.L.} = \frac{2}{3}\,\frac{F_{cu}}{\gamma_c}\, a\, b\left(d - \frac{a}{2}\right) + \frac{F_y}{\gamma_s}\, A_{s`}\,(d - d`)$$

$$\therefore\ M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(138.8)(250)\left(650 - \frac{138.8}{2}\right) + \left(\frac{360}{1.15}\right)(804)(650 - 50)$$

$$= 374865729\ N.mm = 374.8\ kN.m$$

$$\boxed{M_{U.L.} = 374.8\ kN.m}$$

e - The Factor Of Safty For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{374.8}{231.46} = 1.62$$

F - The Factor Of Safty For Material.

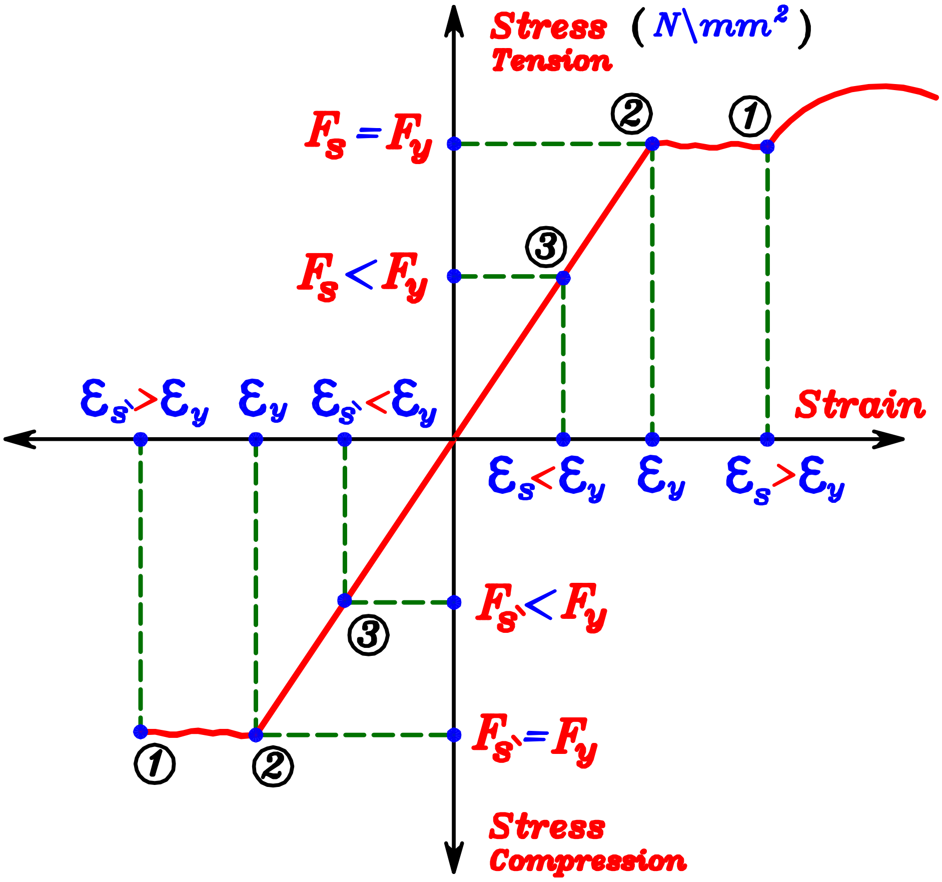
$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{438.2}{374.8} = 1.17$$

g - The Global Factor Of Safty.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{438.2}{231.46} = 1.89$$

شكل الـ Sterss–strain curve للحديد فى الـ Compression

هو نفس شكل الـ Sterss–strain curve للحديد فى الـ Tension



max. stress (Concrete) = F_{cu}

max. stress (Tension Steel) = F_y

max. stress (Compression Steel) = F_y

max. strain (Concrete) = $\varepsilon_c = 0.003$

strain at yield (Tension Steel) $\varepsilon_s = \varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

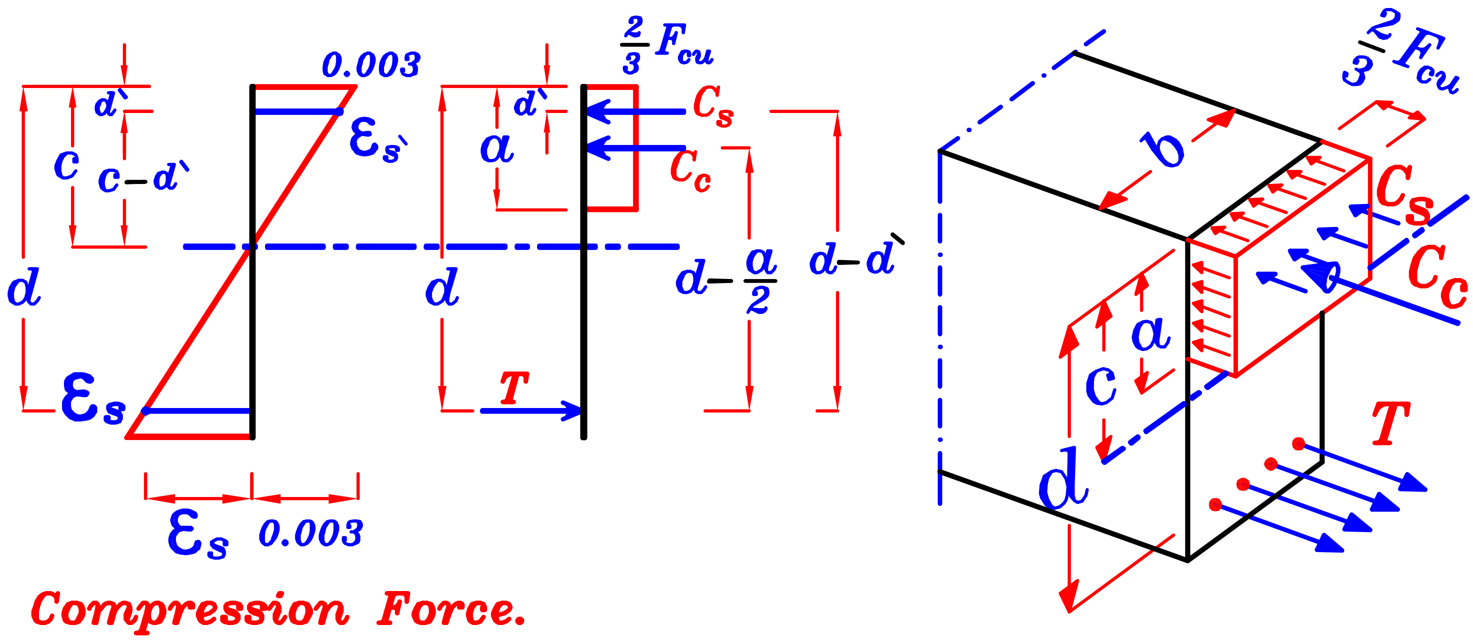
strain at yield (Compression Steel) $\varepsilon_{s'} = \varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

Note.

When $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$

When $\varepsilon_{s'} \geqslant \varepsilon_y \longrightarrow F_{s'} = F_y$

IF there is compressive steel.



Compression Force.

$$C_c = \frac{2}{3} F_{cu} * (a * b)$$

$$C_s = A_{s`} * F_{s`}$$

Tension Force.

$$T = A_s * F_s$$

When

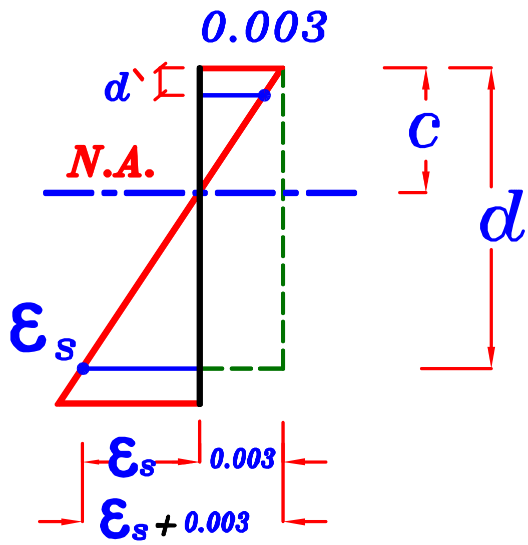
$\varepsilon_s \geq \varepsilon_y \longrightarrow F_s = F_y$

$\varepsilon_{s`} \geq \varepsilon_y \longrightarrow F_{s`} = F_y$

Equilibrium Equation.

$$C_c + C_s = T$$

$$\boxed{\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s}$$

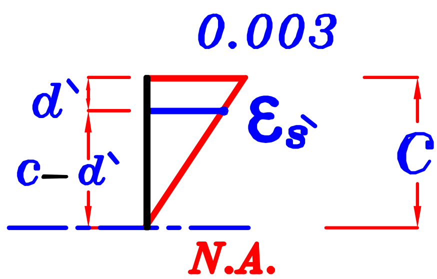


Compatibility Equations.

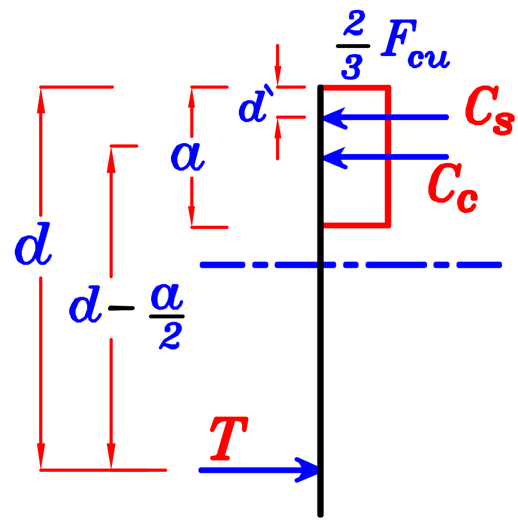
$$\boxed{C = 1.25\, a = \frac{600}{600 + F_s} * d}$$

$$\frac{\varepsilon_{s`}}{0.003} = \frac{c - d`}{c} \qquad \because \varepsilon_{s`} = \frac{F_{s`}}{2 * 10^5}$$

$$\boxed{\frac{F_{s`}}{600} = \frac{c - d`}{c} = \frac{1.25\, a - d`}{1.25\, a}}$$



Steps to determine M_{ult}



① Get $C_b = \dfrac{600}{600+F_y} * d$

② Use equilibrium eqn. $C_c + C_s = T$

$\frac{2}{3}F_{cu} * a * b + A_{s'} * F_{s'} = A_s * F_s$ —— $a, F_s, F_{s'} = ??$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under reinforced or Balanced Sec.)

assume $\varepsilon_{s'} \geqslant \varepsilon_y \longrightarrow F_{s'} = F_y$ Where $\varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

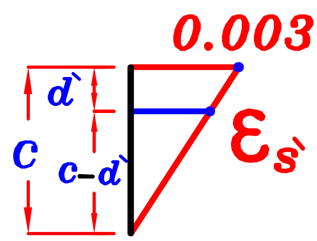
$\therefore \frac{2}{3}F_{cu} * a * b + A_{s'} * F_y = A_s * F_y \longrightarrow$ Get $a \longrightarrow$ Get $c = 1.25\,a$

$*$ IF $c \leqslant c_b$

$\therefore$ The Section is Under reinforced or Balanced Sec. $\therefore F_s = F_y$

To check the second assumption $F_{s'} = F_y$

$\dfrac{\varepsilon_{s'}}{0.003} = \dfrac{c-d'}{c}$ get $\varepsilon_{s'}$

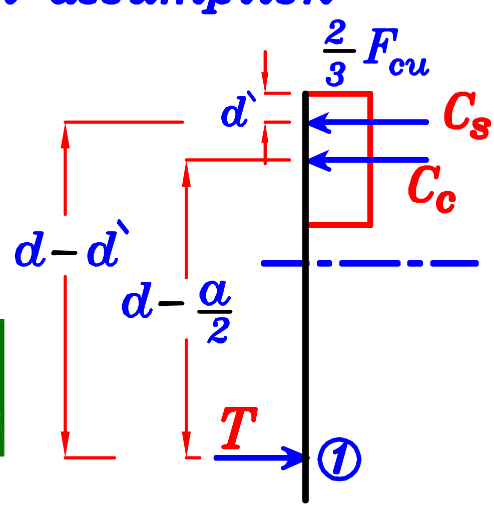


Get $\varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

– IF $\varepsilon_{s'} \geqslant \varepsilon_y \quad \therefore F_{s'} = F_y$ right assumption

$\therefore C_s = A_{s'} F_y$

$, C_c = \frac{2}{3} F_{cu}\, a\, b$



$$\therefore \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b\left(d - \frac{a}{2}\right) + A_{s'} F_y \left(d - d'\right)}$$

$- \; \varepsilon_{s`} < \varepsilon_y \quad \therefore \; F_{s`} < F_y$ wrong assumption

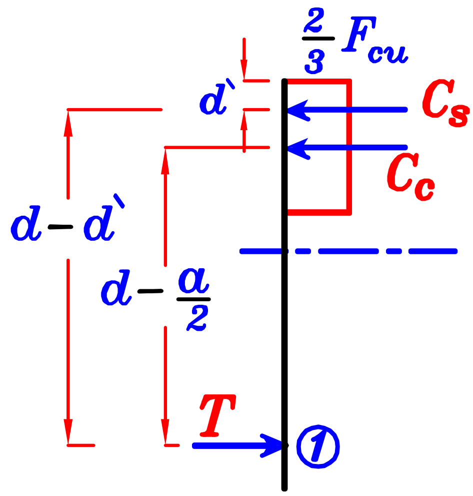
$\therefore$ To get The right value of $a \, , F_{s`}$

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_{s`} = A_s * F_y \qquad \underline{a \, , F_{s`}} \; (1)$$

$$\frac{F_{s`}}{600} = \frac{1.25\,a - d`}{1.25\,a} \qquad \underline{a \, , F_{s`}} \; (2)$$

From eqns. (1) , (2) Get $a \, , F_{s`}$

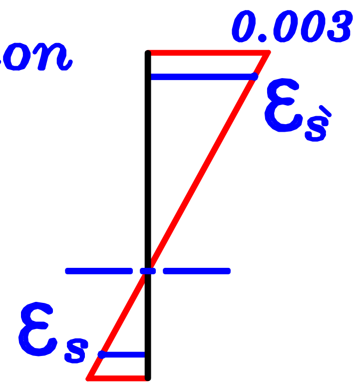
$$\therefore \; \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_{s`} (d - d`)}$$



$*$ IF $\; C > C_b$

$\therefore$ The Section is Over reinforced Sec.

$\therefore \; \varepsilon_s < \varepsilon_y \quad \therefore \; F_s < F_y$ wrong assumption



IF $\; \boxed{\varepsilon_s < \varepsilon_y \longrightarrow \varepsilon_{s`} > \varepsilon_y}$

$\because \; \varepsilon_{s`} > \varepsilon_y \quad \therefore \; F_{s`} = F_y$

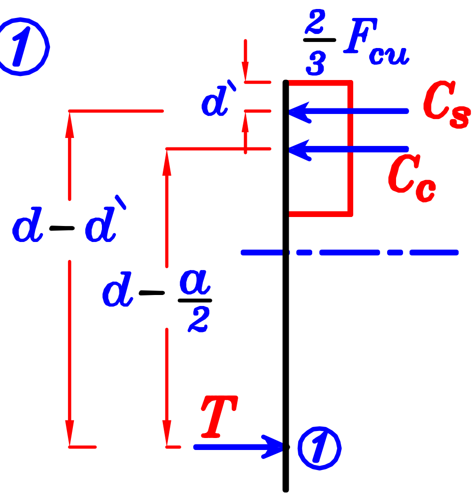
$\therefore$ To get The right value of $a \, , F_s$

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_y = A_s * F_s \qquad \underline{a \, , F_s} \; (1)$$

$$C = 1.25\,a = \frac{600}{600 + F_s} * d \qquad \underline{a \, , F_s} \; (2)$$

From eqns. (1) , (2) Get $a \, , F_s$

$$\therefore \; \boxed{M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)}$$



To Calculate M_{ult}
(With Ten. & comp. Steel)

Get $C_b = \dfrac{600}{600 + F_y} * d$

From equilibrium eqn. $\frac{2}{3} F_{cu} * (a * b) + A_{s'} * F_{s'} = A_s * F_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (The section is under reinforced or Balanced Sec.)

assume $\varepsilon_{s'} \geqslant \varepsilon_y \longrightarrow F_{s'} = F_y$ Where $\varepsilon_y = \dfrac{F_y}{E_s} = \dfrac{F_y}{2*10^5}$

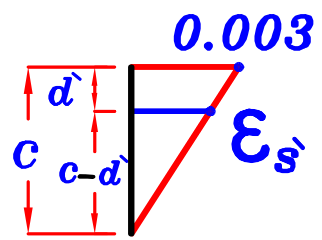
$\therefore \frac{2}{3} F_{cu} * (a * b) + A_{s'} * F_y = A_s * F_y \longrightarrow$ Get $a \longrightarrow$ Get $C = 1.25\, a$

IF $\boxed{C}$

IF $C \leqslant C_b$

Under or Balanced Sec. $\therefore F_s = F_y$

To check $F_{s'} = F_y$



From $\dfrac{\varepsilon_{s'}}{0.003} = \dfrac{c - d'}{c}$ get $\varepsilon_{s'}$

$\varepsilon_{s'} \geqslant \varepsilon_y$

$\therefore F_{s'} = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s'} F_y (d - d')$$

$\varepsilon_{s'} < \varepsilon_y$ $\therefore F_{s'} < F_y$

to get a , $F_{s'}$

$\frac{2}{3} F_{cu} * a * b + A_{s'} * F_{s'} = A_s * F_y$ — a , $F_{s'}$ ①

$\dfrac{F_{s'}}{600} = \dfrac{1.25\, a - d'}{1.25\, a}$ — a , $F_{s'}$ ②

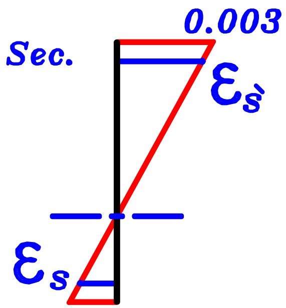
$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s'} F_{s'} (d - d')$$

IF $C > C_b$

Over Reinforced Sec.

$\varepsilon_s < \varepsilon_y \longrightarrow F_s < F_y$

$\therefore \varepsilon_{s'} \geqslant \varepsilon_y \longrightarrow F_{s'} = F_y$



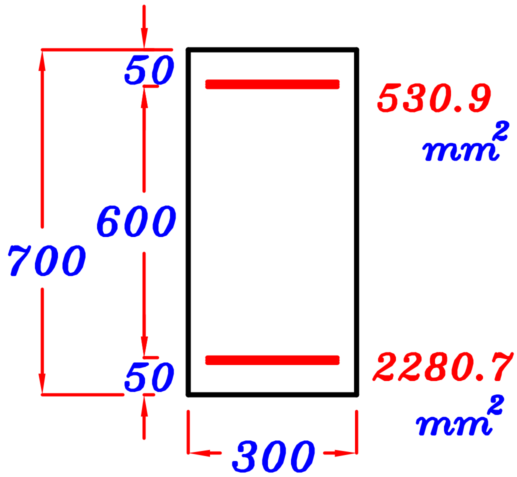
$\frac{2}{3} F_{cu} * a * b + A_{s'} * F_y = A_s * F_s$ — a , F_s ①

$C = 1.25\, a = \dfrac{600}{600 + F_s} * d$ — a , F_s ②

From ①, ② get a , F_s

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s'} F_y (d - d')$$

Example.



Data.

$F_{cu} = 25\ N\backslash mm^2$, st. 360/520

Req.

Calculate $M_{ult.}$

Solution.

$d = 650\ mm$, $d` = 50\ mm$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25\ mm$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

$$\frac{2}{3}(25)(a)(300) + (530.9)(360) = (2280.7)(360)$$

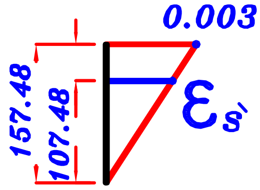
$\longrightarrow a = 125.98\ mm \longrightarrow C = 1.25\ a = 157.48\ mm < C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the First assumption is right $F_s = F_y$

To check if the second assumption is right or wrong. $F_{s`} = F_y$

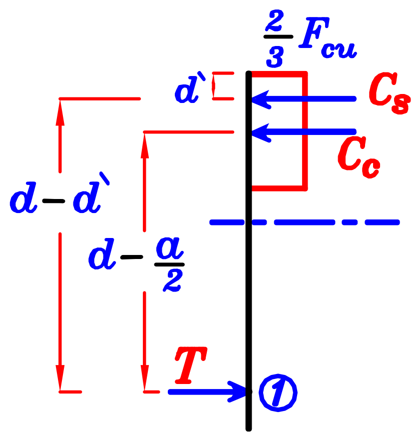
Get $\varepsilon_y = \dfrac{F_y}{2*10^5} = \dfrac{360}{2*10^5} = 1.8 * 10^{-3}$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C} = \dfrac{107.48}{157.48} \longrightarrow \varepsilon_{s`} = 2.047 * 10^{-3}$

$\therefore \varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$ $\therefore$ The second assumption is right.

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, b \left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)$$



$$= \frac{2}{3}(25)(125.98)(300)\left(650 - \frac{125.98}{2}\right) + (530.9)(360)(650 - 50)$$

$= 484431999\ N.mm = 484.43\ kN.m$

$\therefore$ $\boxed{M_{ult} = 484.43\ kN.m}$

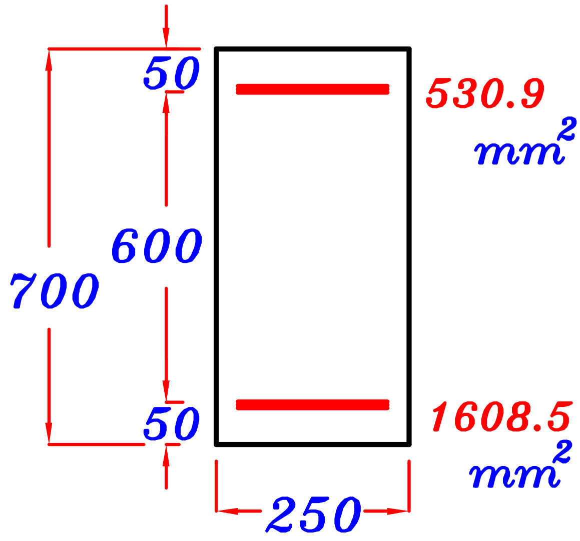
Example.

Data.

$F_{cu} = 25 \ N\backslash mm^2$

st. 360/520

Req. Calculate $M_{ult.}$



Solution.

$d = 650 \ mm$, $d` = 50 \ mm$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600+360} * 650 = 406.25 \ mm$

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

$$\frac{2}{3} (25)(a)(250) + (530.9)(360) = (1608.5)(360)$$

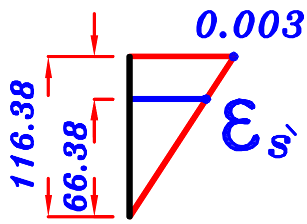
$\longrightarrow a = 93.1 \ mm \longrightarrow C = 1.25 \ a = 116.38 \ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the First assumption is right $F_s = F_y$

To check if the second assumption is right or wrong. $F_{s`} = F_y$

Get $\varepsilon_y = \dfrac{F_y}{2*10^5} = \dfrac{360}{2*10^5} = 1.8 * 10^{-3}$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C-d`}{C} = \dfrac{66.38}{116.38} \longrightarrow \varepsilon_{s`} = 1.711 * 10^{-3}$

$\therefore \varepsilon_{s`} < \varepsilon_y \longrightarrow F_{s`} < F_y \therefore$ The second assumption is wrong.

To Get the right value of $a, F_{s'}$

* From equilibrium eqn.

$$\frac{2}{3} F_{cu} * a * b + A_{s'} * F_{s'} = A_s * F_y$$

$$\frac{2}{3}(25)(a)(250) + (530.9)(F_{s'}) = (1608.5)(360)$$

$$F_{s'} = 1090.71 - 7.848 \; a \qquad \underline{a, F_{s'}} \; ①$$

* From compatibility eqn.

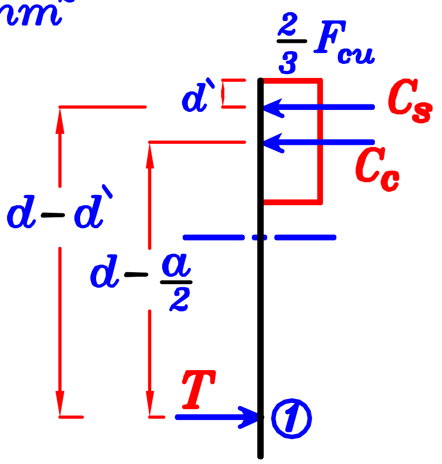
$$\frac{F_{s'}}{600} = \frac{1.25 a - d'}{1.25 a} \qquad \underline{a, F_{s'}} \; ②$$

From eqns. ①, ②

$$\frac{(1090.71 - 7.848 a)}{600} = \frac{1.25 a - 50}{1.25 a} \longrightarrow a = 94.78 \; mm$$

$$, F_{s'} = 1090.71 - 7.848(94.78) = 346.87 \; N\backslash mm^2$$

$$\therefore M_{ult} = \frac{2}{3} F_{cu} \, a \, b \left(d - \frac{a}{2}\right) + A_{s'} F_{s'} (d - d')$$



$$\therefore M_{ult} = \frac{2}{3}(25)(94.78)(250)\left(650 - \frac{94.78}{2}\right) + (530.9)(346.87)(650 - 50)$$

$$= 348472702 \; N.mm = 348.47 \; kN.m$$

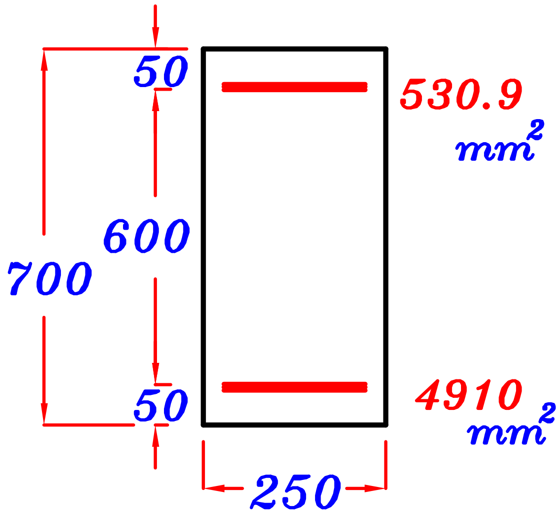
$$\therefore \boxed{M_{ult} = 348.47 \; kN.m}$$

Example.

Data.

$F_{cu} = 25\ N\backslash mm^2$ st. 360/520

Req. Calculate $M_{ult.}$



Solution. $d = 650\ mm$, $d` = 50\ mm$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25\ mm$

② From equilibrium eqn. $C_c + C_s = T$

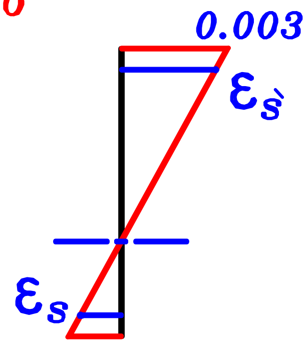
$$\frac{2}{3} F_{cu} * (a * b) + A_{s`} * F_{s`} = A_s * F_s$$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = F_y$ (Under or Balanced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = F_y$

$$\frac{2}{3}(25)(a)(250) + (530.9)(360) = (4910)(360)$$

$\longrightarrow a = 378.35\ mm \longrightarrow C = 1.25\ a = 472.94\ mm > C_b$



$\therefore$ $\boxed{\text{The Section is Over Reinforced Sec.}}$

and the First assumption is wrong $F_s < F_y$

But the second assumption will be right $F_{s`} = F_y$

To Get the right value of a , F_s

$$\frac{2}{3}(25)(a)(250) + (530.9)(360) = (4910)\ F_s \quad a , F_s \text{ -------- ①}$$

$$1.25 a = \frac{600}{600 + F_s} * 650 \quad \text{-------------------------} \quad a , F_s \text{ --- ②}$$

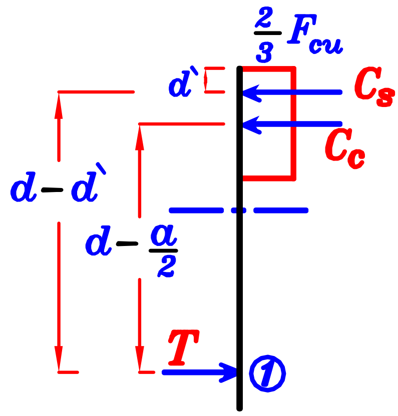
$a = 337.31\ mm$

$F_s = 324.96\ N\backslash mm^2$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\ a\ b\left(d - \frac{a}{2}\right) + A_{s`} F_y (d - d`)$$

$$= \frac{2}{3}(25)(337.31)(250)\left(650 - \frac{337.31}{2}\right) + (530.9)(360)(650 - 50)$$

$= 791184741\ N.mm = 791.18\ kN.m$

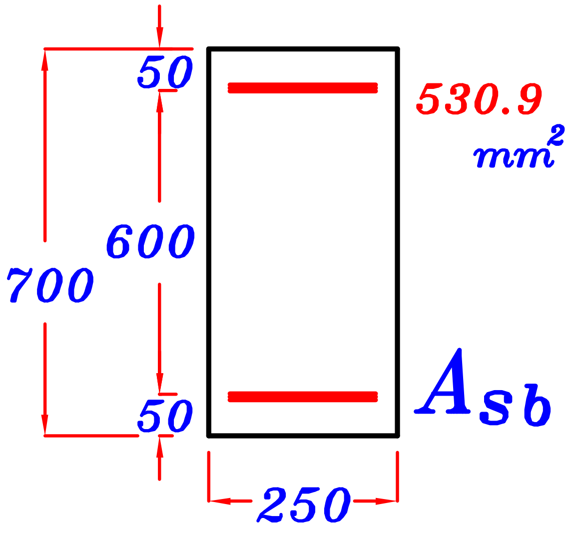


$\therefore$ $\boxed{M_{ult} = 791.18\ \ kN.m}$

Example.

Data. $F_{cu} = 25 \ N\backslash mm^2$

st. 360/520



Req. Calculate A_{sb}

To make the sec. is balanced Sec.

and then get M_b

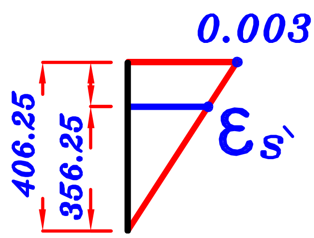
Solution.

For Balanced Sec. $C = C_b \ , \ a = a_b = 0.8\, C_b \ , \ F_s = F_y$

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 650 = 406.25 \ mm$

② $a = a_b = 0.8\, C_b = 0.8 * 406.25 = 325 \ mm$

③ Get $\varepsilon_y = \dfrac{F_y}{2*10^5} = \dfrac{360}{2*10^5} = 1.8 * 10^{-3}$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C} = \dfrac{356.25}{406.25} \longrightarrow \varepsilon_{s`} = 2.63 * 10^{-3}$

$\therefore \ \varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = F_y$

④ From equilibrium eqn. $C_c + C_s = T$

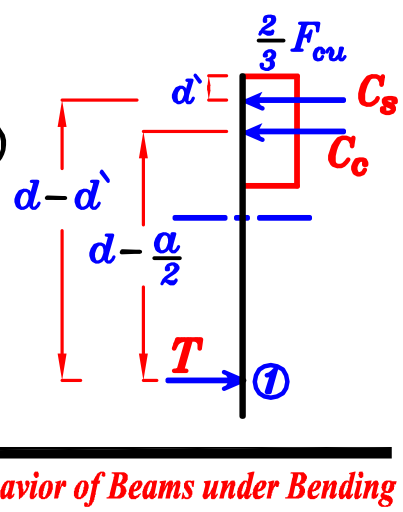
$$\frac{2}{3} F_{cu} * (a_b * b) + A_{s`} * F_y = A_{sb} * F_y$$

$$\frac{2}{3}(25)(325)(250) + (530.9)(360) = A_{sb}(360) \quad \therefore \quad \boxed{A_{sb} = 4292.4 \ mm^2}$$

$$\therefore \ M_{ult} = \frac{2}{3} F_{cu} \, a_b \, b \left(d - \frac{a_b}{2}\right) + A_{s`} F_y (d - d`)$$

$$M_{ult} = \frac{2}{3}(25)(325)(250)\left(650 - \frac{325}{2}\right) + (530.9)(360)(650 - 50)$$

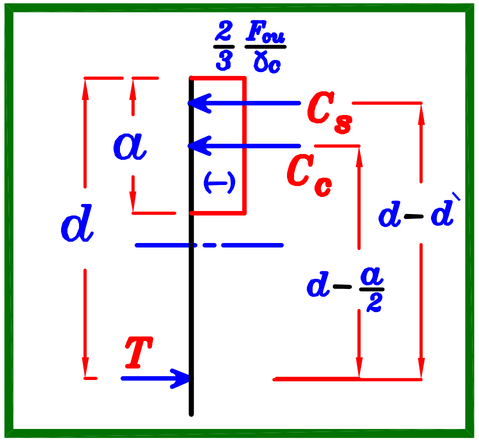
$$M_{ult} = 774830650 \ N.mm = 774.83 \ kN.m$$



$$\therefore \ \boxed{M_{ult} = 774.83 \ kN.m}$$

To Calculate $M_{U.L.}$ (With Ten. & Comp.Steel)

Calculate $a_{max} = \frac{2}{3} C_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600+(F_y \backslash \delta_s)}\right] * d$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * F_{s`} = A_s * F_s$

assume $\varepsilon_s \geqslant \varepsilon_y \longrightarrow F_s = \frac{F_y}{\delta_s}$ (Under reinforced Sec.)

assume $\varepsilon_{s`} \geqslant \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\delta_s}$ where $\varepsilon_y = \frac{(F_y \backslash \delta_s)}{2*10^6}$

$\therefore \frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_s * \frac{F_y}{\delta_s} = A_{s`} * \frac{F_y}{\delta_s} \longrightarrow$ Get a

IF $\boxed{a}$

IF $a \leqslant 0.1\,d$

take $a = 0.1\,d$, neglect $A_{s`}$

because $F_{s`}$ is very small.

$\therefore M_{U.L.} = A_s \frac{F_y}{\delta_s}\left(d - \frac{a}{2}\right)$

$\therefore M_{U.L.} = A_s \frac{F_y}{\delta_s}\left(d - \frac{0.1d}{2}\right)$

$\therefore M_{U.L.} = A_s F_y\, d \,\frac{1}{1.15}\left(1 - \frac{0.1}{2}\right)$

$\therefore \boxed{M_{U.L.} = 0.826\, A_s\, F_y\, d}$

$0.1\,d < a < a_{max.}$

The First assumption is right $F_s = \frac{F_y}{\delta_s}$

To check $F_{s`} = \frac{F_y}{\delta_s}$ From $\frac{\varepsilon_{s`}}{0.003} = \frac{c-d`}{c}$

$\boxed{\varepsilon_{s`}}$

IF $\varepsilon_{s`} \geqslant \varepsilon_y \quad \therefore F_{s`} = \left(\frac{F_y}{\delta_s}\right)$

$$\boxed{M_{ult} = \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) a\, b\left(d - \frac{a}{2}\right) + A_{s`}\left(\frac{F_y}{\delta_s}\right)(d - d`)}$$

IF $\varepsilon_{s`} < \varepsilon_y \quad \therefore F_{s`} < \left(\frac{F_y}{\delta_s}\right)$

To Get $a, F_{s`}$

$\frac{2}{3}\frac{F_{cu}}{\delta_c} * a * b + A_{s`} * F_{s`} = A_s * \frac{F_y}{\delta_s}$ —— $a, F_{s`}$ (1)

$\frac{F_{s`}}{600} = \frac{1.25\,a - d`}{1.25\,a}$ —— $a, F_{s`}$ (2)

$$\therefore \boxed{M_{U.L.} = \frac{2}{3}\left(\frac{F_{cu}}{\delta_c}\right) a\, b\left(d - \frac{a}{2}\right) + A_{s`} F_{s`}(d - d`)}$$

IF $a > a_{max.}$

Take $a = a_{max.}$, $F_{s`} = \frac{F_y}{\delta_s}$

$$\therefore \boxed{M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\delta_c} a_{max.} b\left(d - \frac{a_{max.}}{2}\right) + A_{s`}\left(\frac{F_y}{\delta_s}\right)(d - d`)}$$

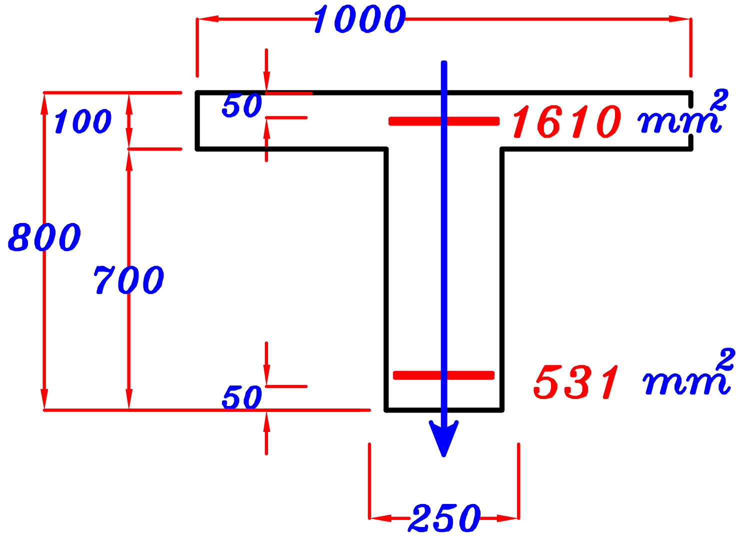
Example.

Data.

$F_{cu} = 25 \ N \backslash mm^2$

st. $360/520$

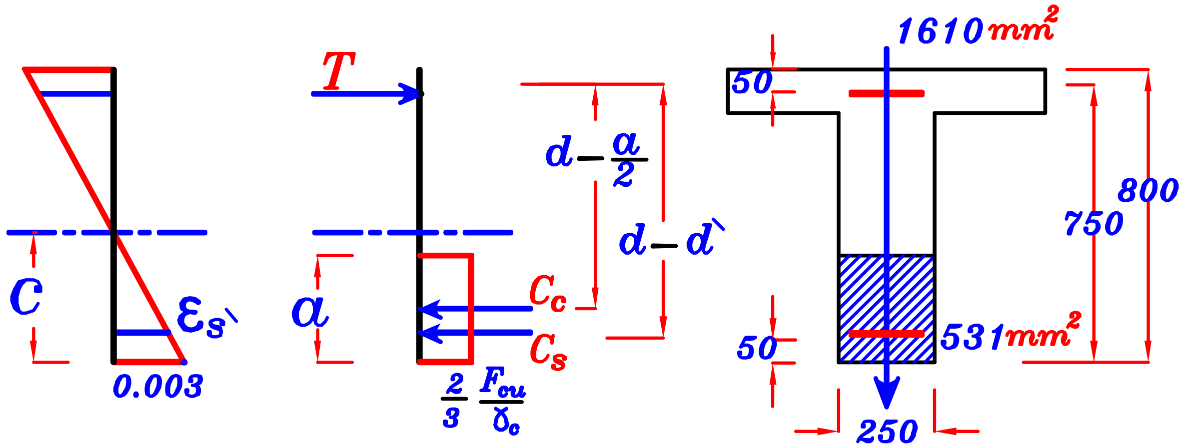
Req. Calculate $M_{U.L.}$



Solution.

$0.1\,d = 75 \ mm$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.35\,d = 0.35 * 750 = 262.5 \ mm$$



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b + A_{s`} * F_{s`} = A_s * F_s$

assume $\varepsilon_s > \varepsilon_y \longrightarrow F_s = \frac{F_y}{\gamma_s}$ (**Under reinforced Sec.**)

assume $\varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = \frac{F_y}{\gamma_s}$

$$\therefore \ \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * b + A_{s`} * \frac{F_y}{\gamma_s} = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)(250) + (531)\left(\frac{360}{1.15}\right) = (1610)\left(\frac{360}{1.15}\right)$$

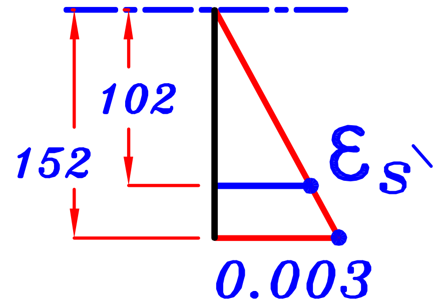
$$\longrightarrow a = 121.6 \ mm$$

$\therefore\ 0.1\,d < a < a_{max.}$ **Right assumption** $F_s = \dfrac{F_y}{\gamma_s}$

To check IF $F_{s`} = \dfrac{F_y}{\gamma_s}$ **or not**

Get $\varepsilon_y = \dfrac{F_y/\gamma_s}{E_s} = \dfrac{360/1.15}{2*10^5} = 1.565*10^{-3}$

$C = 1.25\,a = 1.25 * 121.6 = 152\ mm$



From $\dfrac{\varepsilon_{s`}}{0.003} = \dfrac{C - d`}{C}$

$\therefore\ \dfrac{\varepsilon_{s`}}{0.003} = \dfrac{102}{152} \longrightarrow \varepsilon_{s`} = 2.013*10^{-3}$

$\therefore\ \varepsilon_{s`} > \varepsilon_y \longrightarrow F_{s`} = \dfrac{F_y}{\gamma_s}$

$$\therefore\ M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c}\,a\,b\left(d - \frac{a}{2}\right) + A_{s`}\frac{F_y}{\gamma_s}(d - d`)$$

$$\therefore\ M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(121.6)(250)\left(750 - \frac{121.6}{2}\right) + (531)\left(\frac{360}{1.15}\right)(750 - 50)$$

$= 349154705\ N.mm = 349.15\ kN.m$

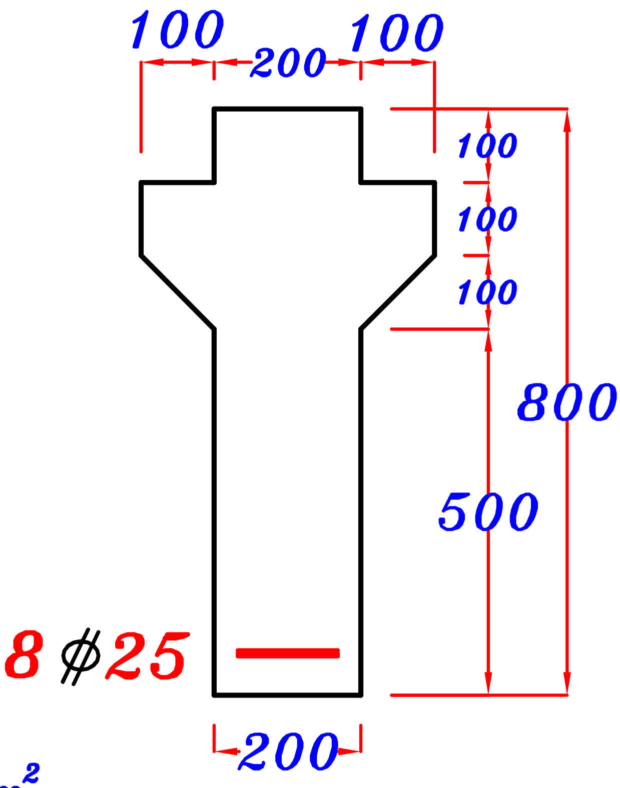
$$\boxed{M_{U.L.} = 349.15\ kN.m}$$

Example.

Data.

$F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$



Req.

Calculate M_w

Solution.

$$A_s = 8\,\phi 25 = 8\left[\frac{\pi * 25^2}{4}\right] = 3927\ mm^2$$

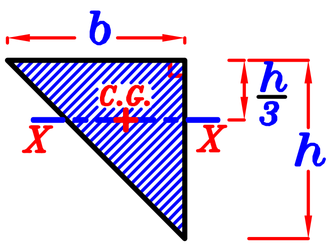
Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

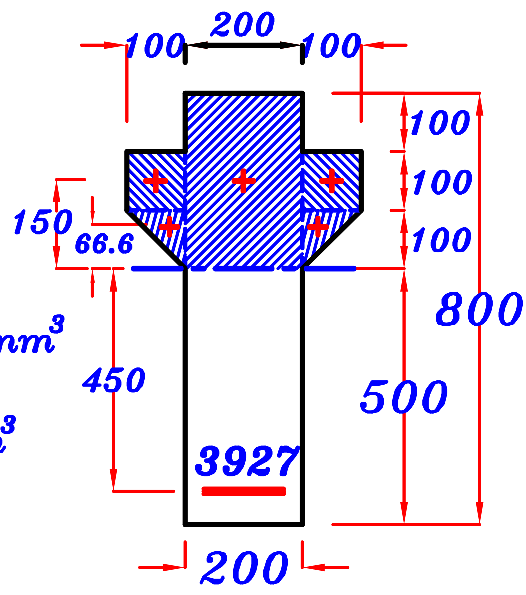
$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

Inertia For right angle Triangle

$$\boxed{I_x = \frac{b\,h^3}{36}}$$



To know if Z is bigger or smaller than 300 mm

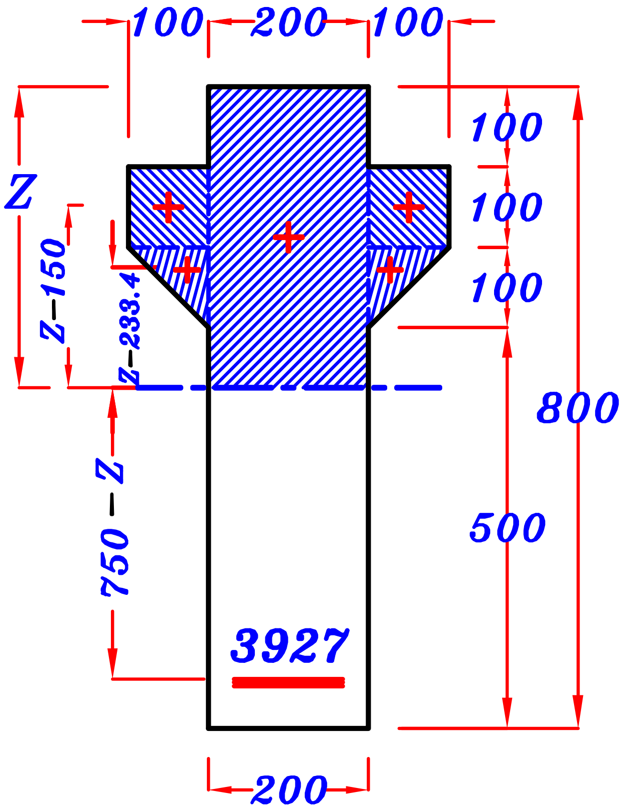


$$S_{nv.}(\text{above}) = (200)(300)(150) + 2\,(100)(100)(150) + 2\left(\frac{1}{2}\right)(100)(100)(66.6) = 12666000\ mm^3$$

$$S_{nv.}(\text{under}) = 15 * 3927 * (450) = 26507250\ mm^3$$

$\because\ S_{nv.}(\text{under}) > S_{nv.}(\text{above})$

$\therefore\ Z > 300\ mm$



① Take $n = 15$

② Get Z by taking $S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}$

$$200(Z)\left(\frac{Z}{2}\right) + 2(100)(100)(Z-150)$$

$$+\,2\left(\frac{1}{2}\right)(100)(100)(Z-233.4)$$

$$= (15)(3927)(750-Z)$$

$$\boxed{Z = 387.77\ mm}$$

③ Get $I_{nv} = \frac{200(387.77)^3}{3} + 2\left(\frac{100*100^3}{12}\right) + 2(100)(100)(387.77-150)^2$

$$+\,2\left(\frac{100*100^3}{36}\right) + 2\left(\frac{1}{2}\right)(100)(100)(387.77-233.34)^2$$

$$+\,(15)(3927)(750-387.77)^2 = 13007509270\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 13007509270}{387.77} = 318671733.5\ N.mm$

$= 318.67\ kN.m$

⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 13007509270}{750-387.77} = 478793741.5\ N.mm$

$= 478.7\ kN.m$

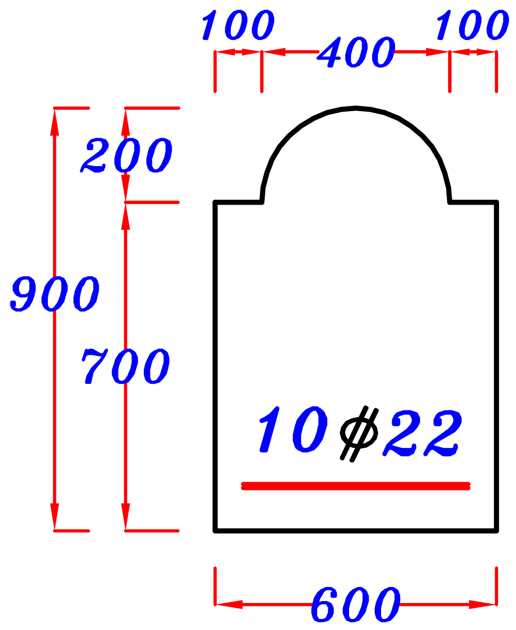
⑥ $\boxed{M_w = 318.67\ kN.m}$

Example.

Data.

$F_{cu} = 25 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$



Req.

Calculate M_w

Solution.

$$A_s = 10\#22 = 10\left[\frac{\pi * 22^2}{4}\right] = 3801 \ mm^2$$

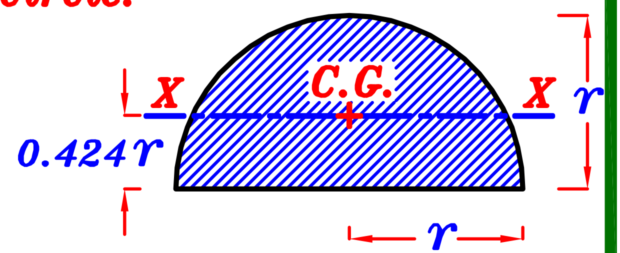
Allowable stresses

$F_{cu} = 25 \quad N\backslash mm^2 \longrightarrow F_c = 9.5 \quad N\backslash mm^2$

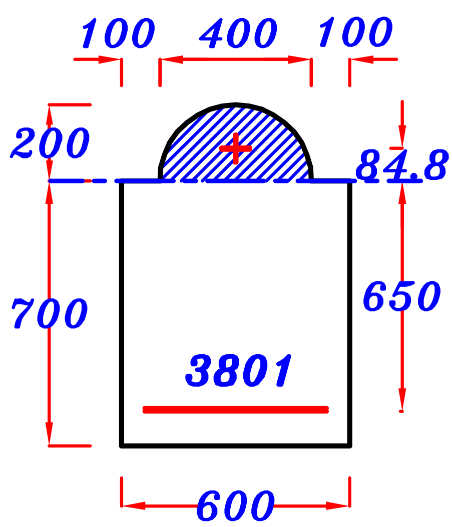
$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$

Inertia For semi circle.

$$\boxed{I_x = 0.11\, r^4}$$



To know if Z is bigger or smaller than 200 mm

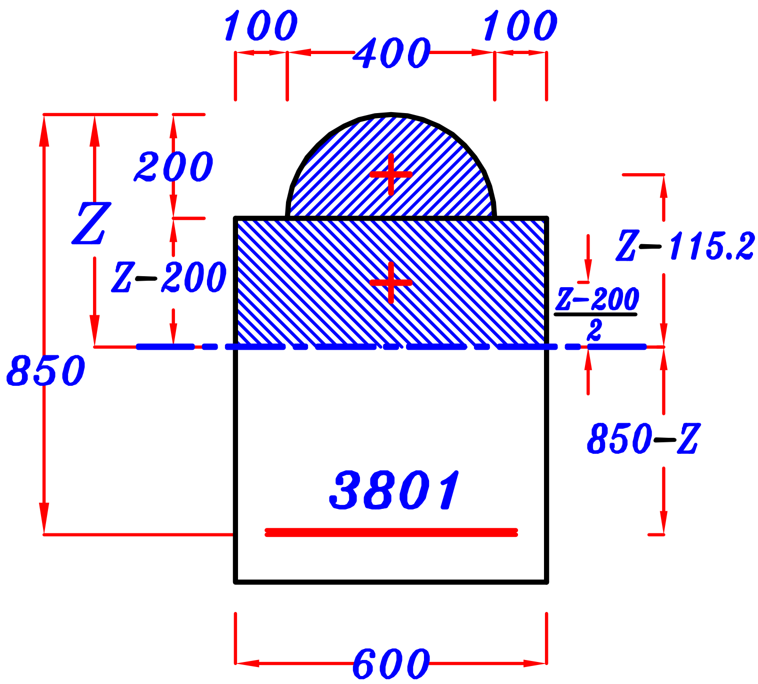


$$S_{nv.}(\text{above}) = \frac{\pi\,(200)^2}{2}(84.8) = 5328141.1 \ mm^3$$

$$S_{nv.}(\text{under}) = 15 * 3800 * (650) = 37059750 \quad mm^3$$

$\because \ S_{nv.}(\text{under}) > S_{nv.}(\text{above})$

$\therefore \ Z > 200 \ mm$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$\frac{\pi\,(200)^2}{2}\,(Z-115.2) + (600)\,(Z-200)\left(\frac{Z-200}{2}\right)$$

$$= (15)\,(3801)\,(850-Z) \qquad \boxed{Z = 381.92\ mm}$$

③ Get $I_{nv} = 0.11\,(200)^4 + \frac{\pi\,(200)^2}{2}\,(381.92-115.2)^2$

$$+ \frac{600 * 181.89^3}{3} + (15)\,(3801)\,(850-381.92)^2$$

$$= 18341282030\ mm^4$$

④ $M_{wc} = \frac{F_c * I_{nv}}{Z} = \frac{9.5 * 18341282030}{381.92} = 456226904\ N.mm$
$= 456.22\ kN.m$

⑤ $M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z} = \frac{\left(\frac{200}{15}\right) * 18341282030}{850-381.92} = 522454339\ N.mm$
$= 522.45\ kN.m$

⑥ $\boxed{M_w = 456.22\ kN.m}$

Example.

For the reinforced concrete cross-section shown in the Figure
It is required to calculate :

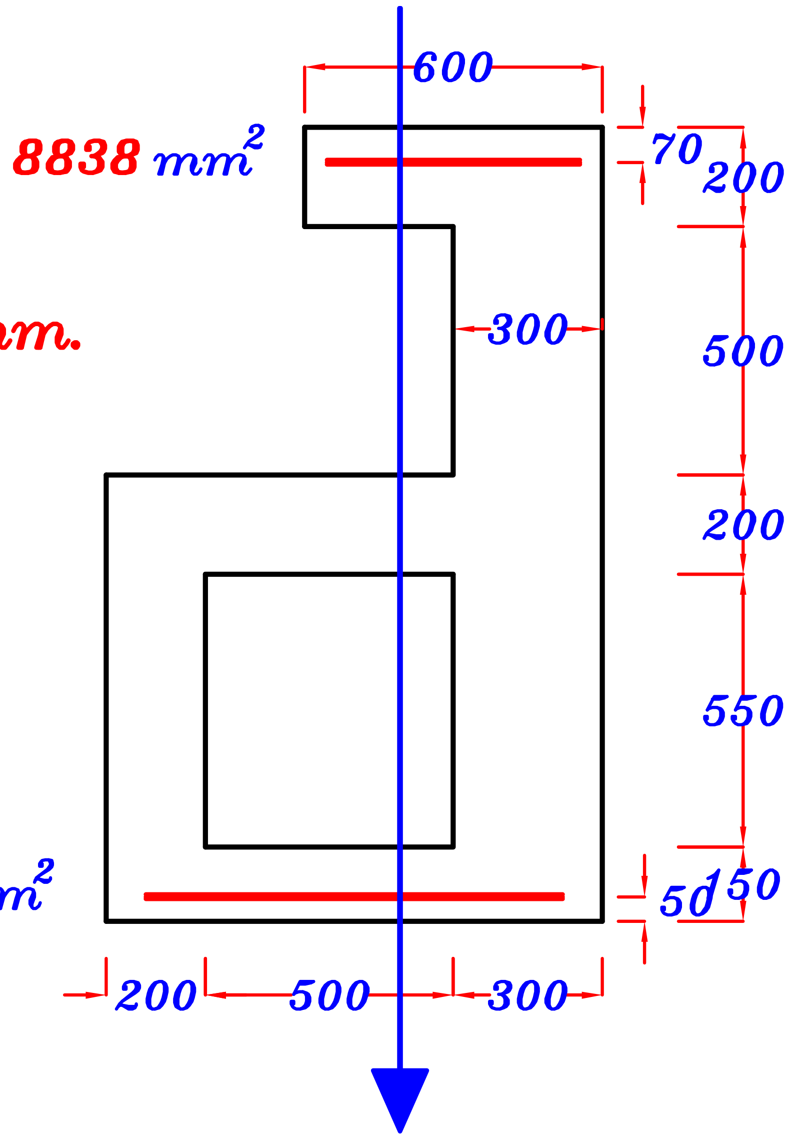
1 - Calculate the cracking moment $(M_{cr.})$, the working moment (M_w), the ultimate limit moment $(M_{U.L.})$ & the ultimate moment $(M_{ult.})$

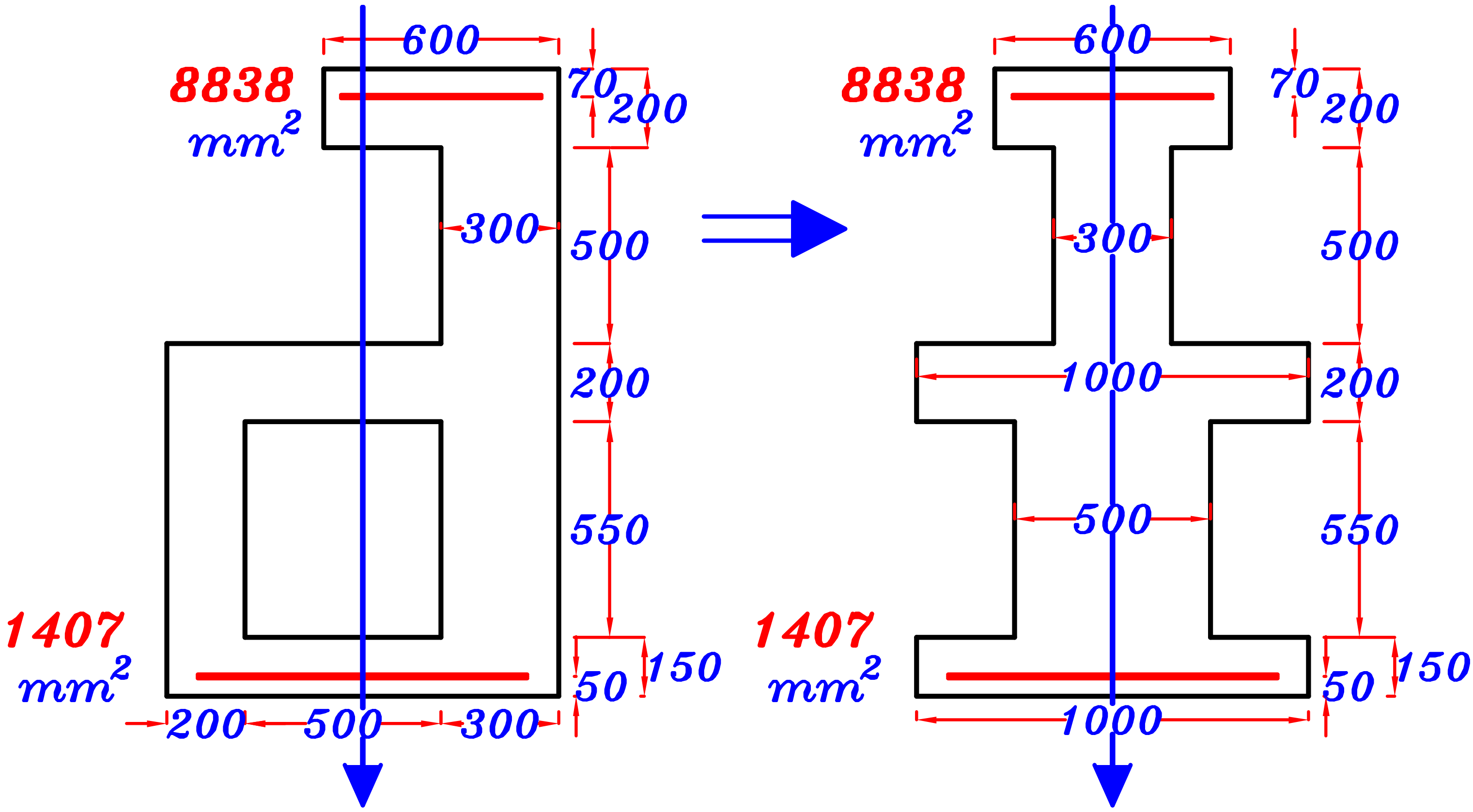
2 - Calculate the Factors of safety For Loads, Materials & Global Factor of safety.

Data : $F_{cu} = 25\ N\backslash mm^2$

, st. 400/600

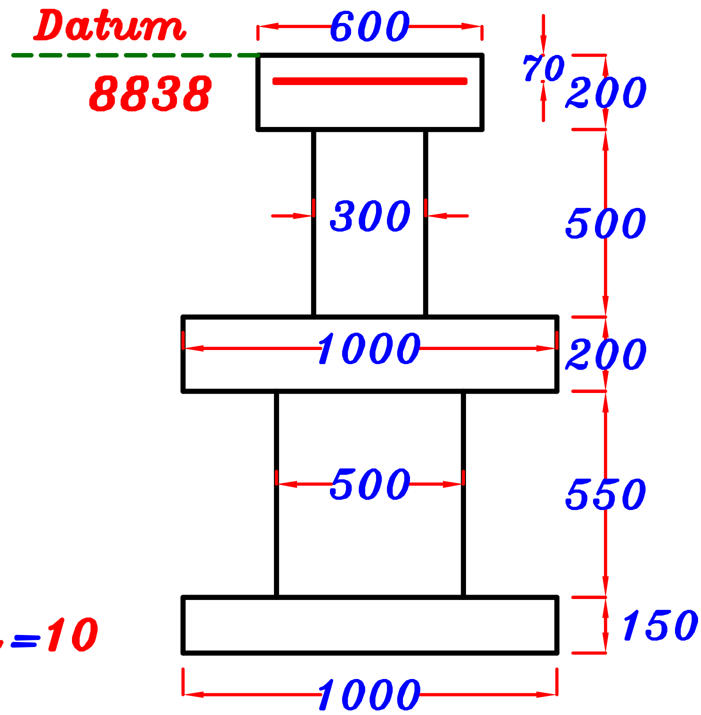
All dimensions in mm.





$$\frac{A_s'}{A_s} = \frac{1407}{8838} = 0.159 < 0.2$$

We can neglect A_s'



a- The Cracking Moment. ($M_{cr.}$)

① $n = \dfrac{E_s}{E_c} = \dfrac{2 * 10^5}{4400\sqrt{25}} = 9.1 \longrightarrow n = 10$

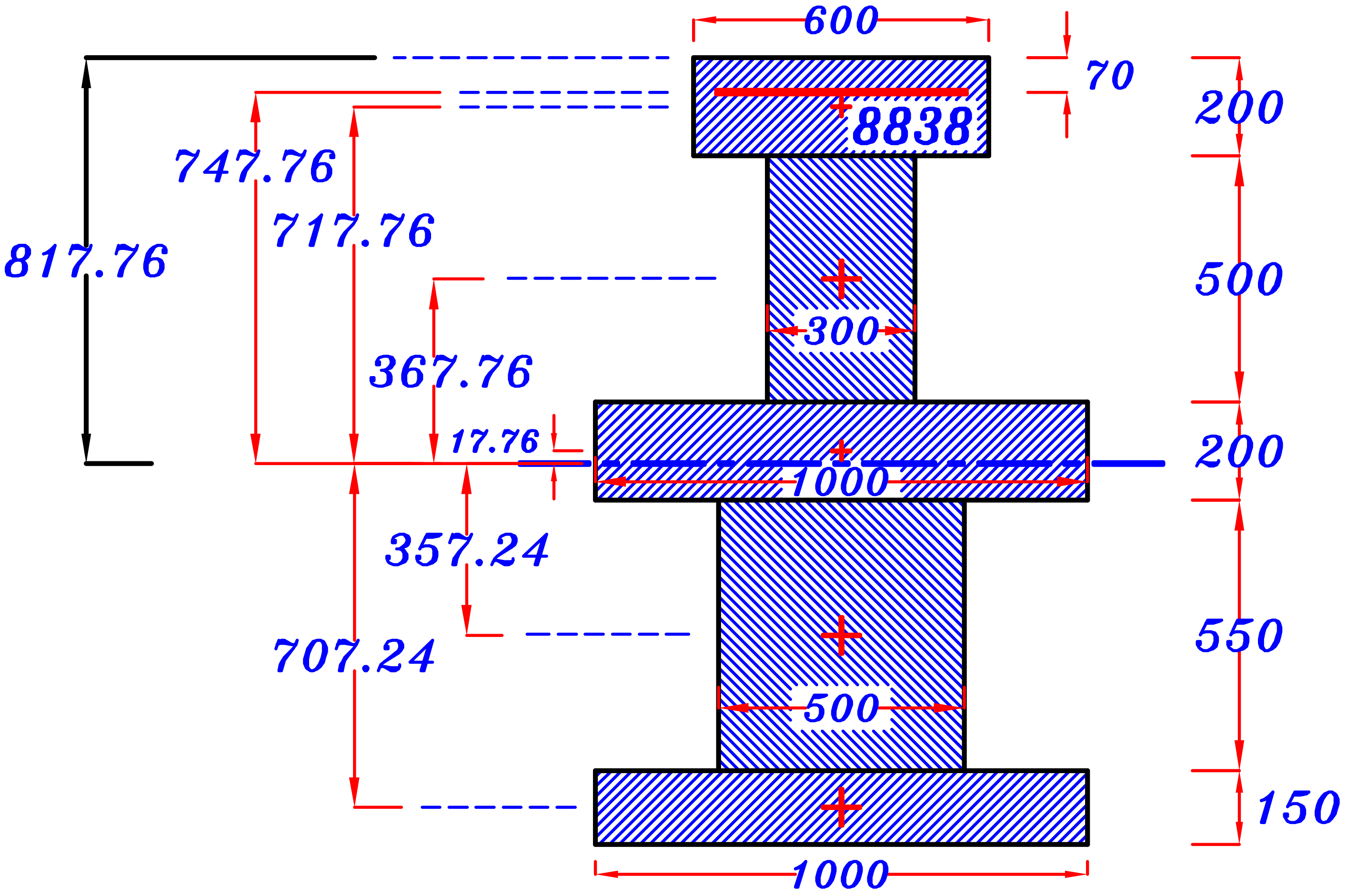
② $A_v = A_c + (n-1) A_s$

$$A_v = 600*200 + 300*500 + 1000*200 + 500*550 + 1000*150 + (10-1)(8838)$$

$$= 974542 \; mm^2$$

③ $$\bar{y}_t = \frac{600*200(100) + 300*500(450) + 1000*200(800) + 500*550(1175) + 1000*150(1525) + (10-1)(8838)(70)}{974542}$$

$$= 817.76 \; mm$$



④ $$I_g = \frac{600*200^3}{12} + 600*200\,(717.76)^2 + \frac{300*500^3}{12} + 300*500\,(367.76)^2 + \frac{1000*200^3}{12}$$

$$+1000*200\,(17.76)^2 + \frac{500*550^3}{12} + 500*550\,(357.24)^2 + \frac{1000*150^3}{12} + 1000*150\,(707.24)^2$$

$$+(10-1)(8838)(747.76)^2 = 248176325100 \text{ mm}^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0 \quad N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\overline{y_t}} = \frac{3.0 * 248176325100}{817.76} = 910449245.8 \text{ N.mm}$$

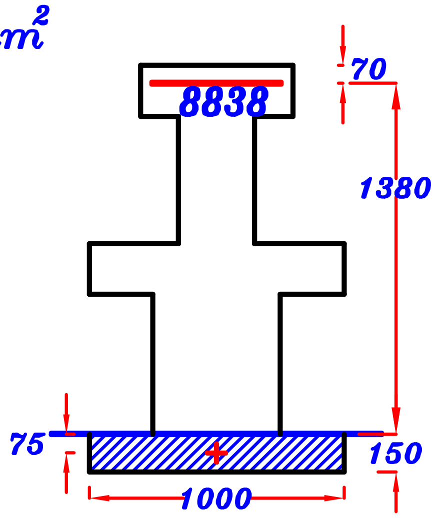
$$= 910.45 \text{ kN.m.}$$

$$\boxed{M_{cr} = 910.45 \text{ kN.m}}$$

b – The Working Moment. (M_w)

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.50 \ N\backslash mm^2$

$F_y = 400 \ N\backslash mm^2 \longrightarrow F_s = 220 \ N\backslash mm^2$

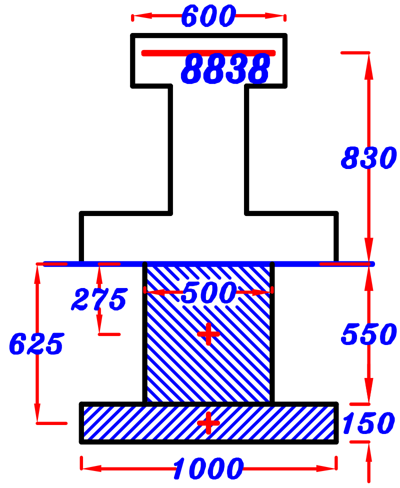


To know if Z bigger or smaller than 150 mm assume First that $Z = 150$ mm

$S_{nv.}(under) = 1000 * 150 * (75) = 11250000 \ mm^3$

$S_{nv.}(above) = 15 * 8838 * (1380) = 182946600 \ mm^3$

$\because \ S_{nv.}(above) > S_{nv.}(under) \ \therefore \ Z > 150$ mm



To know if Z bigger or smaller than 700 mm assume First that $Z = 700$ mm

$S_{nv.}(under) = 1000 * 150 * (625) + 500 * 550 * (275)$

$= 169375000 \ mm^3$

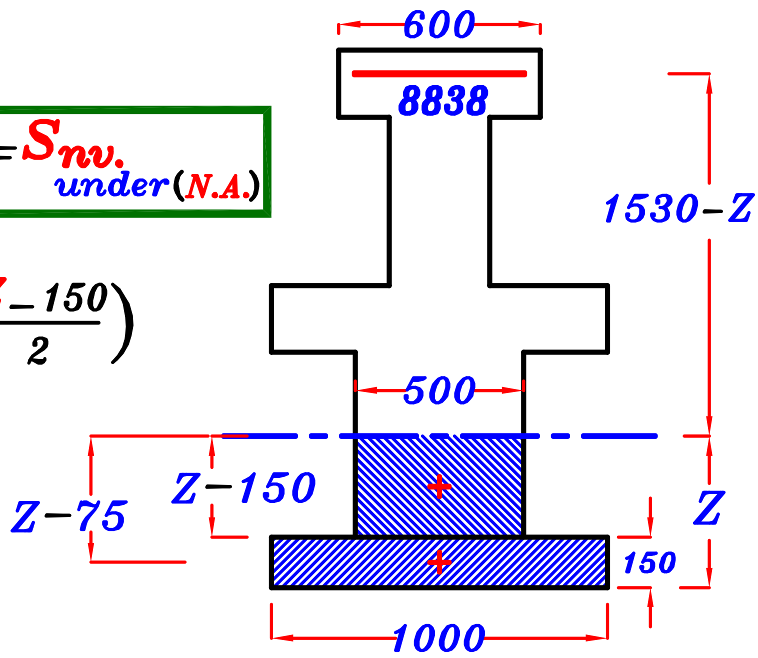
$S_{nv.}(above) = 15 * 8838 * (830) = 110033100 \ mm^3$

$\because \ S_{nv.}(above) < S_{nv.}(under) \ \therefore \ Z < 700$ mm

$\therefore \ \boxed{150 \ mm < Z < 700 \ mm}$

1. Take $n = 15$

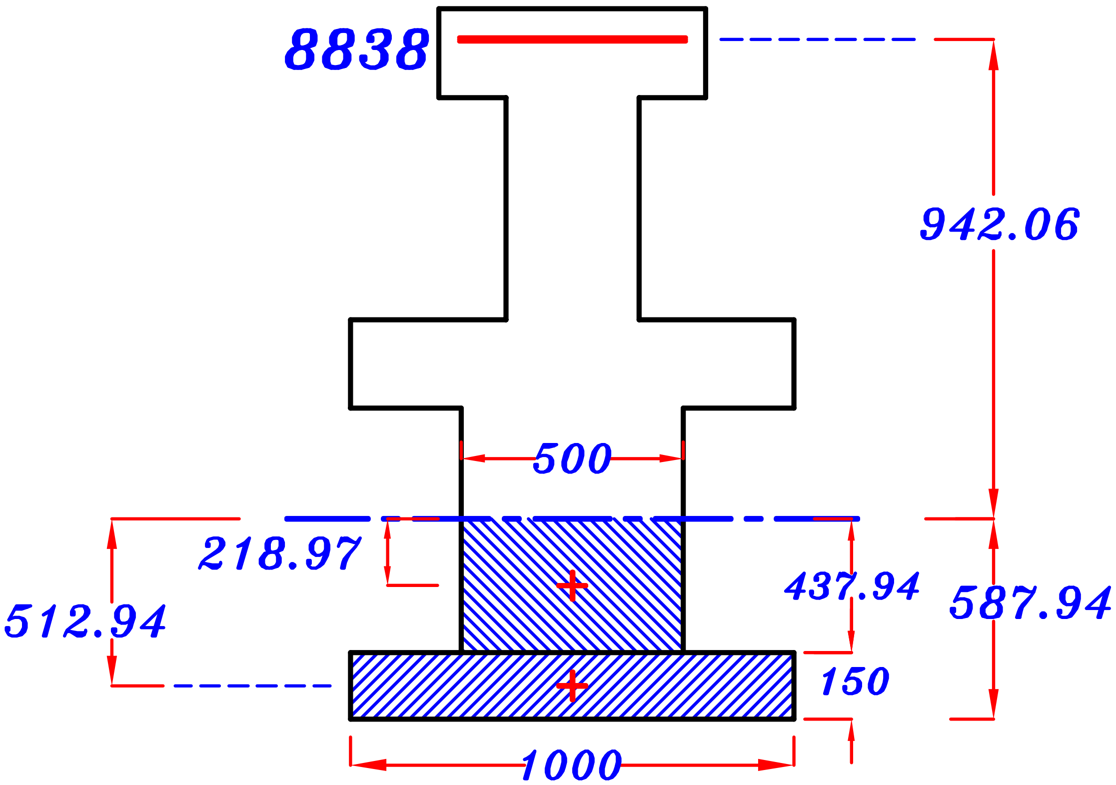
2. Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$



$$(1000)(150)(Z-75) + (500)(Z-150)\left(\frac{Z-150}{2}\right)$$

$$= (15)(8838)(1530-Z)$$

$$\boxed{Z = 587.94 \ mm}$$



③ $$I_{nv} = \frac{1000(150)^3}{12} + (1000)(150)(512.94)^2 + \frac{500(437.94)^3}{3}$$

$$+ (15)(8838)(942.06)^2 = 171399055700 \ mm^4$$

④ $$M_{wc} = \frac{F_c * I_{nv}}{Z}$$ ----- not as T_Sec.

$$= \frac{9.5 * 171399055700}{587.94} = 2769485031 \ N.mm$$

$$= 2769.48 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \frac{\left(\frac{220}{15}\right) * 171399055700}{1530 - 587.94} = 2668463597 \ N.mm$$

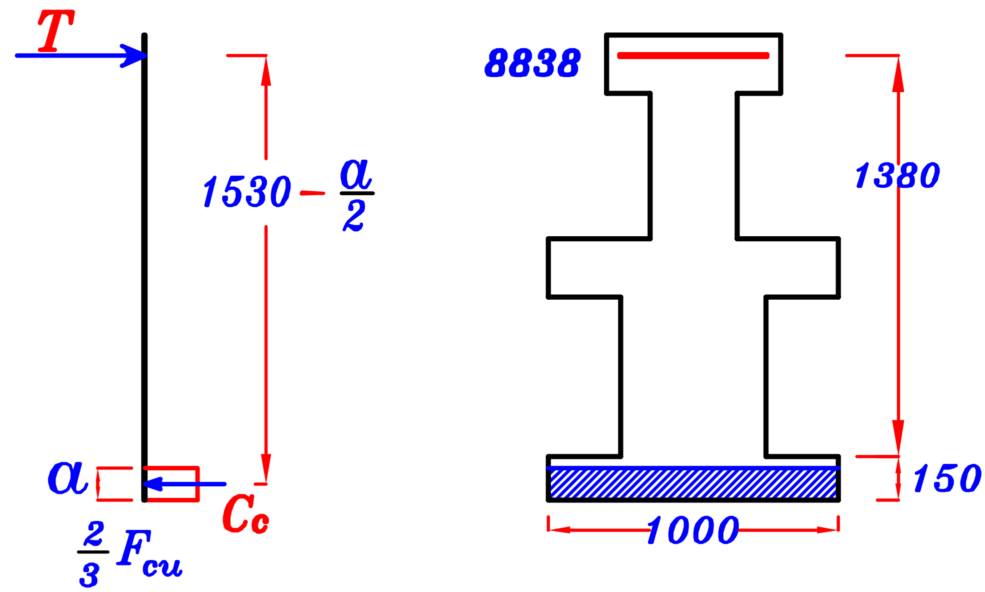
$$= 2668.46 \ kN.m$$

⑥ $$\boxed{M_w = 2668.46 \ kN.m}$$

c – The Failure Moment. (M_{ult})

① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 400} * 1530 = 918\ mm$

② Assume $a < 150\ mm$



③ From equilibrium eqn. $C_c = T$

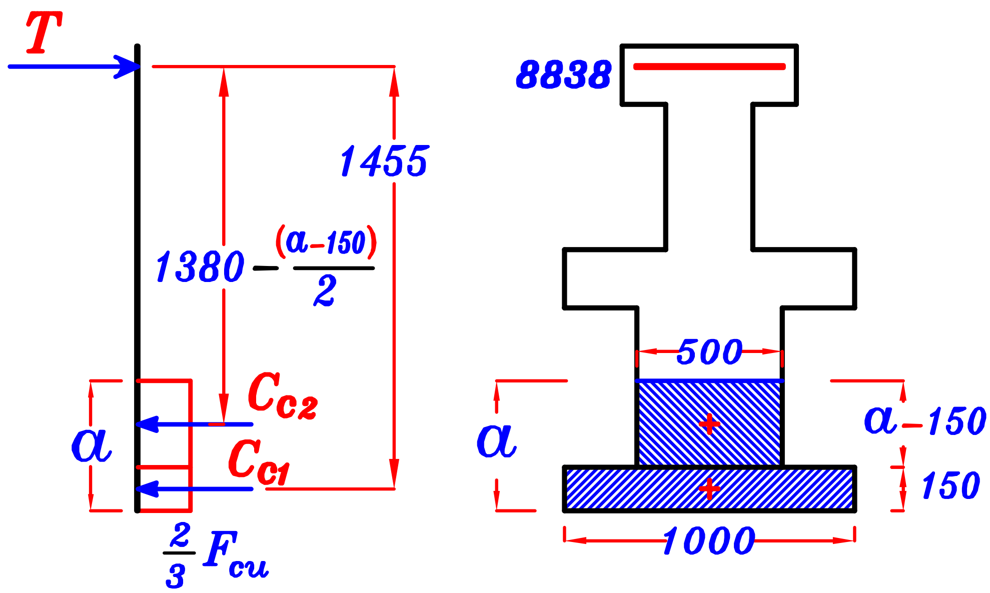
$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3} (25)(a)(1000) = (8838)(400)$$

$\therefore\ a = 212.1\ mm > 150\ mm \quad \therefore$ wrong assumption

$\therefore\ a > 150\ mm$



③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * (1000 * 150) + \frac{2}{3} F_{cu} * [500\,(a - 150)] = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (*under reinforced or Balanced Sec.*)

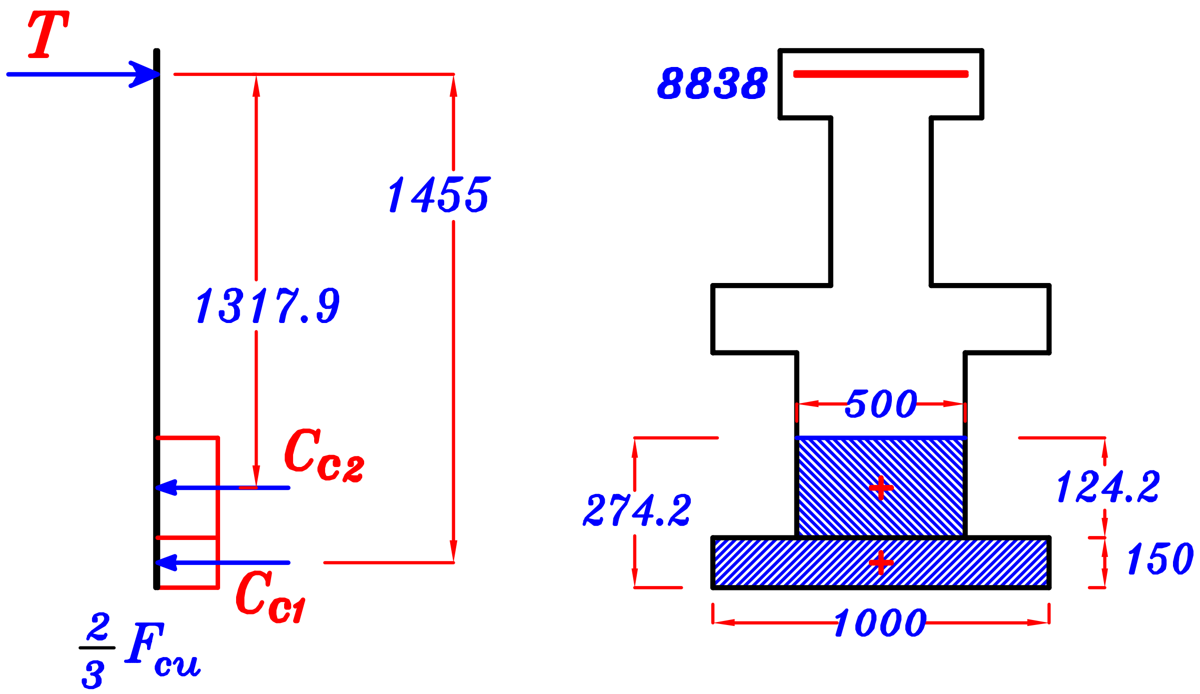
$$\therefore \frac{2}{3}(25)(1000 * 150) + \frac{2}{3}(25) * [500\,(a - 150)] = 8838 * 400$$

$\therefore a = 274.2\ mm > 150\ mm$ $\quad\therefore$ right assumption.

$\therefore C = 1.25\, a = 1.25 * 274.2 = 342.75\ mm < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$



$$\therefore M_{ult} = \frac{2}{3}(25)(1000)(150)\,(1455) + \frac{2}{3}(25)(500)(124.2)\,(1317.9)$$

$$= 5001526500\ N.mm$$

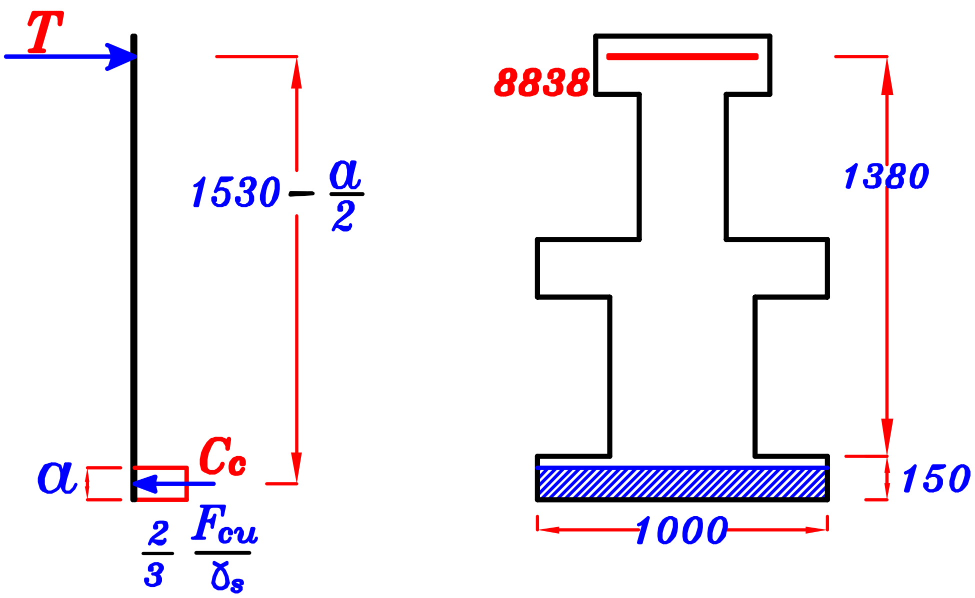
$$\therefore \boxed{M_{ult} = 5001.5\ \ kN.m}$$

d- The Ultimate Limit Moment. ($M_{U.L.}$)

$$a_{min} = 0.1\, d = 0.1 * 1530 = 153.0 \text{ mm}$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.337\, d = 0.337 * 1530 = 515.61 \text{ mm}$$

assume $a < 150$ mm



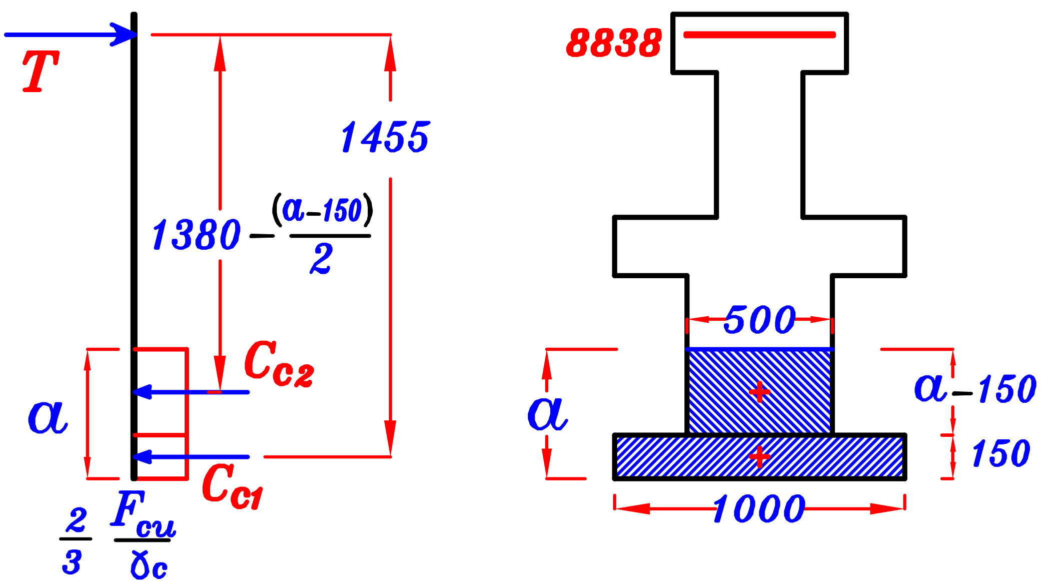
From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * F_s$ ---- a , F_s

$F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.) $\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * \frac{F_y}{\gamma_s}$

$$\frac{2}{3}\left(\frac{25}{1.5}\right)(a)\,(1000) = (8838)\left(\frac{400}{1.15}\right)$$

$\longrightarrow a = 276.6$ mm $> t_s$ $\therefore$ wrong assumption

$\therefore a > 150$ mm



From equilibrium eqn. $C_c = T$

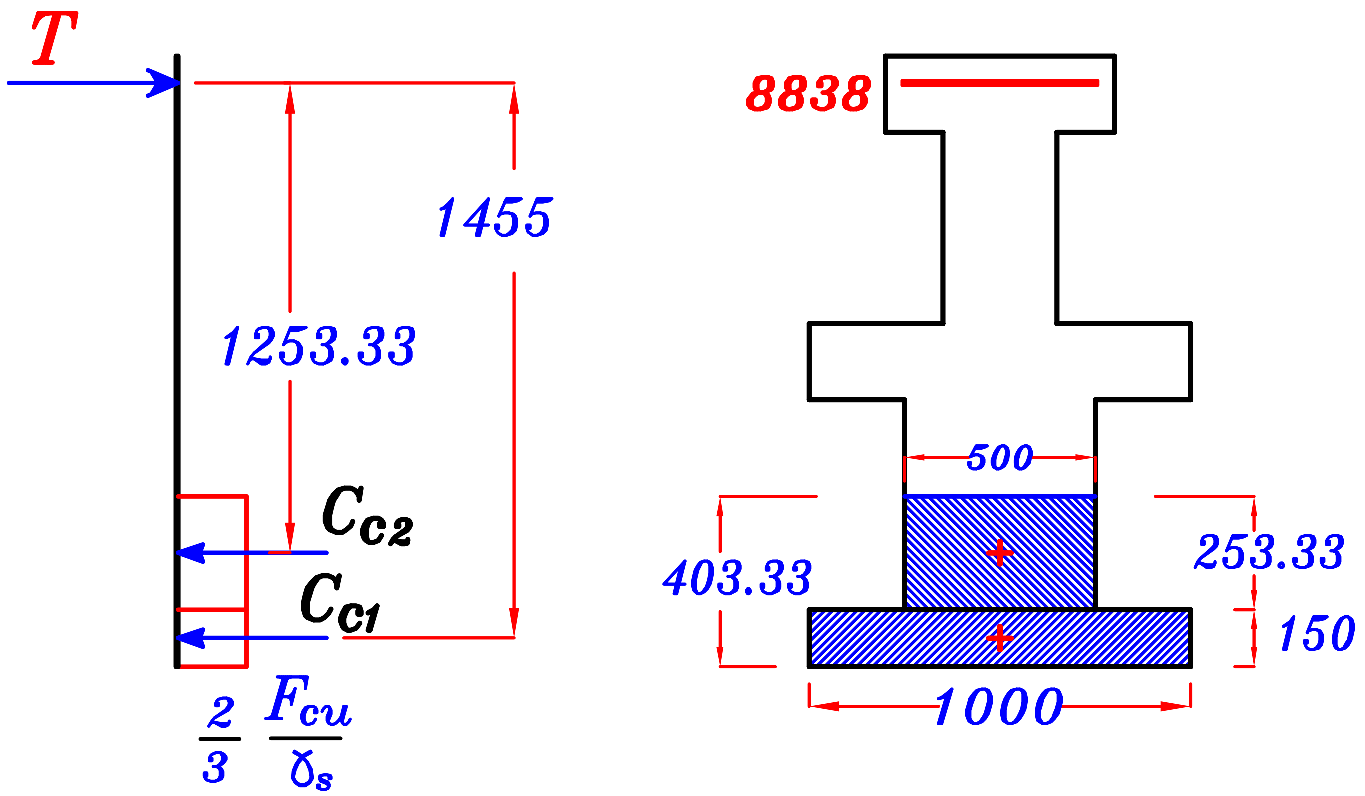
$$\frac{2}{3}\frac{F_{cu}}{\gamma_c} * (1000 * 150) + \frac{2}{3}\frac{F_{cu}}{\gamma_c} * [500\,(a - 150)] = A_s * F_s$$

Assume $F_s = \frac{F_y}{\gamma_s}$ $\longrightarrow$ (under reinforced Sec.)

$$\therefore \frac{2}{3}\left(\frac{25}{1.5}\right)(1000 * 150) + \frac{2}{3}\left(\frac{25}{1.5}\right) * [500\,(a - 150)] = 8838 * \left(\frac{400}{1.15}\right)$$

$\therefore a = 403.33$ mm $>$ 150 mm $\therefore$ right assumption.

$\therefore a_{min} < a < a_{max}$ $\therefore$ right assumption $F_s = \frac{F_y}{\gamma_s}$



$$\therefore M_{U.L.} = \frac{2}{3}\left(\frac{25}{1.5}\right)(1000)(150)(1455) + \frac{2}{3}\left(\frac{25}{1.5}\right)(500)(253.33)(1253.33)$$

$$= 4188922716 \text{ N.mm}$$

$$\therefore \boxed{M_{U.L.} = 4188.92 \text{ kN.m}}$$

- The Factor Of Safty For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{4188.92}{2668.46} = 1.57$$

- The Factor Of Safty For Material.

$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{5001.5}{4188.92} = 1.193$$

- The Global Factor Of Safty.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{5001.5}{2668.46} = 1.87$$

Example.

For the reinforced concrete girder's cross–section shown in **the Figure**
It is required to:

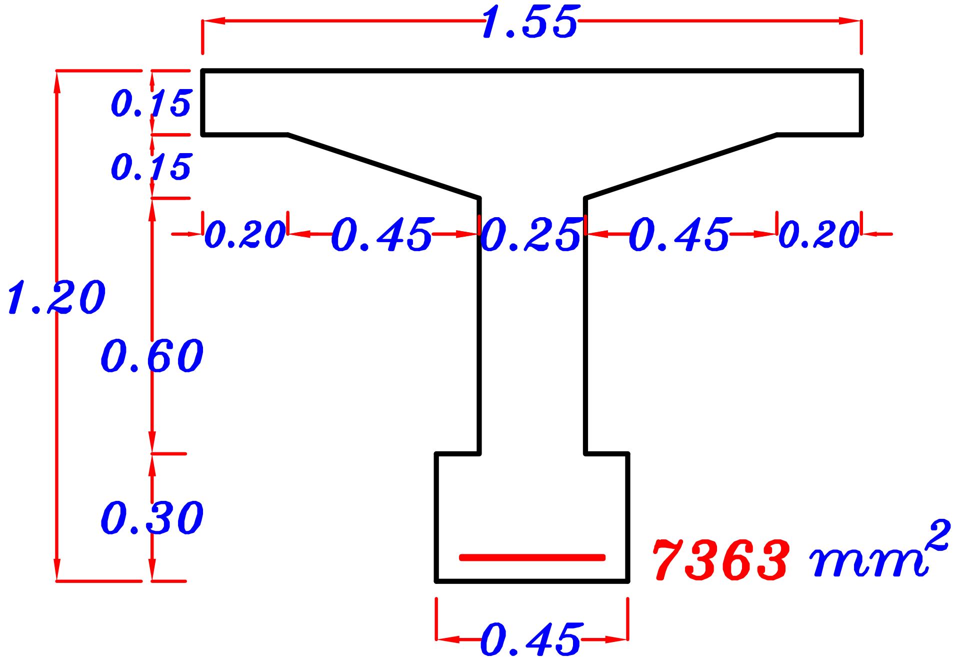
1 – Calculate the cracking moment $(M_{cr.})$, the working moment (M_w), the ultimate limit moment $(M_{U.L.})$ & the ultimate moment $(M_{ult.})$

2 – Calculate the Factors of safety For Loads, Materials & Global Factor of safety.

Data :

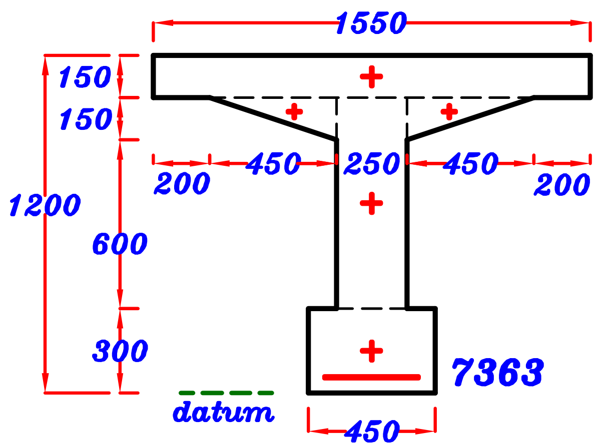
$F_{cu} = 30 \ N\backslash mm^2$

, st. 400/600



---

a– The Cracking Moment. $(M_{cr.})$



① $n = \dfrac{E_s}{E_c} = \dfrac{2 * 10^5}{4400\sqrt{30}} = 8.30 \longrightarrow n = 10$

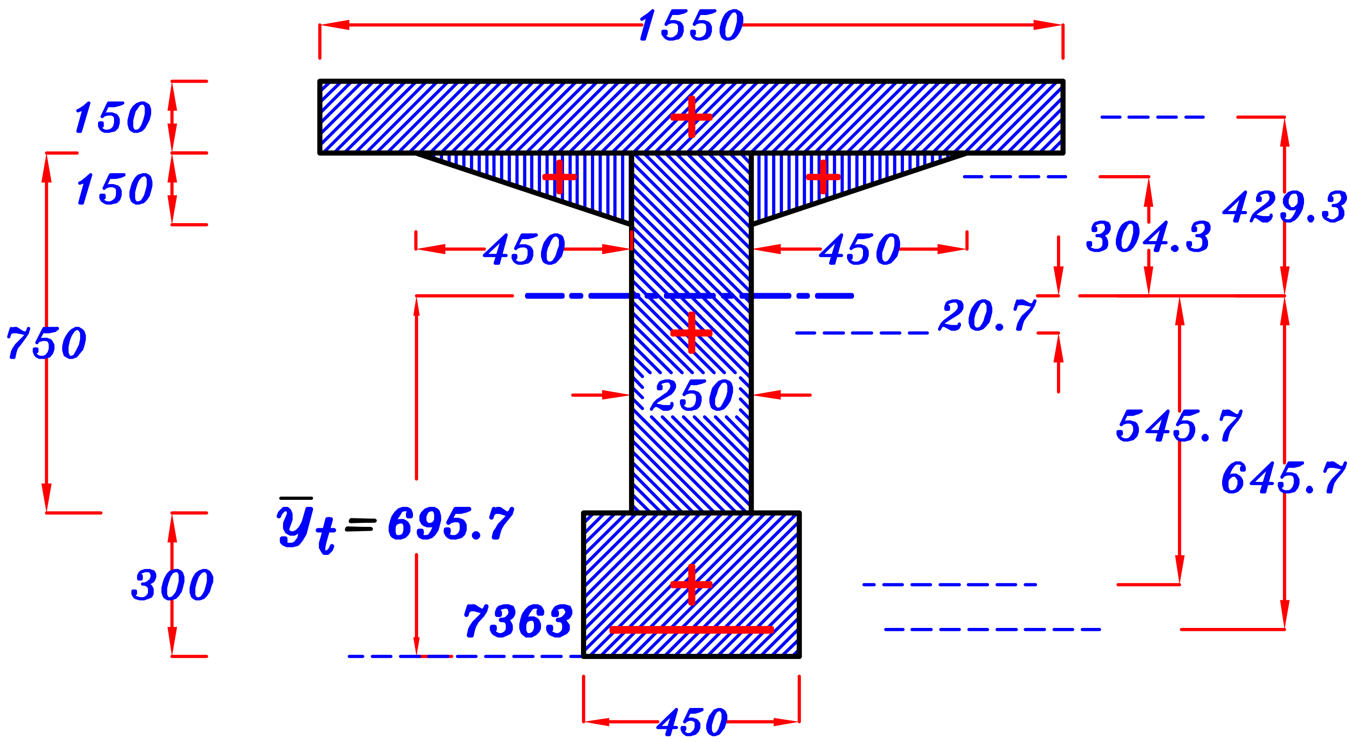
② $A_v = A_c + (n - 1) A_s$

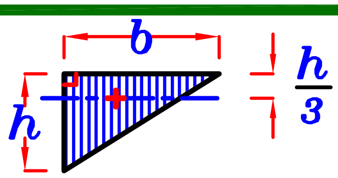
$A_v = 150 * 1550 + 250 * 750 + 2\,(0.5 * 150 * 450)$

$+ 300 * 450 + (10 - 1)(7363) = 688767 \ mm^2$

③ $\bar{y}_t = \dfrac{1550*150\,(1125) + 250*750\,(675) + 2\,(0.5*150*450)\,(1000) + 300*450\,(150) + (10-1)(7363)(50)}{688767}$

$= 695.7 \ mm$





④ $I_g = \frac{1550*150^3}{12} + 1550*150\,(429.3)^2 + 2*\frac{450*150^3}{36} + 2*(0.5*450*150)\,(304.3)^2$

$+ \frac{250*750^3}{12} + 250*750\,(20.7)^2 + \frac{450*300^3}{12} + 450*300\,(545.7)^2$

$+ (10-1)(7363)(645.7)^2 = 127332060300 \; mm^4$

⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28 \; N\backslash mm^2$

⑥ $M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.28 * 127332060300}{695.7} = 600329391.5 \; N.mm$

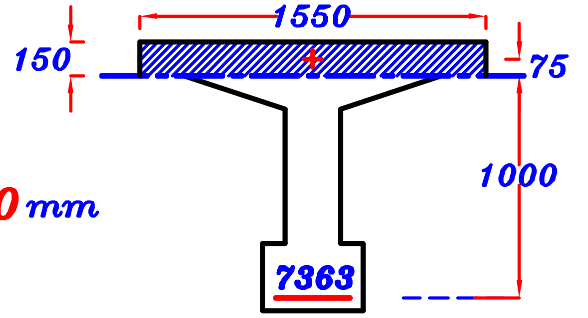
$= 600.32 \; kN.m.$

$$\boxed{M_{cr} = 600.33 \; kN.m}$$

b – The Working Moment. (M_w)

$F_{cu} = 30 \; N\backslash mm^2 \longrightarrow F_c = 10.5 \; N\backslash mm^2$

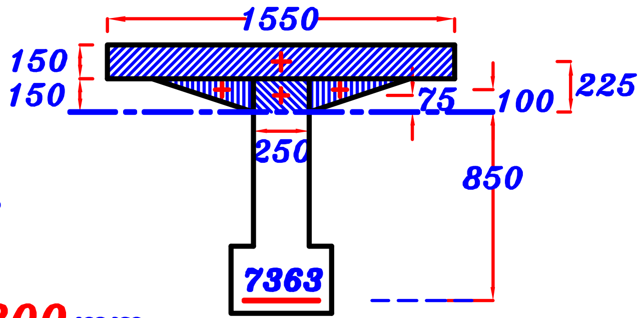
$F_y = 400 \; N\backslash mm^2 \longrightarrow F_s = 220 \; N\backslash mm^2$



$S_{nv.}(above) = 150 * 1550 * (75) = 17437500 \; mm^3$

$S_{nv.}(under) = 15 * 7363 * (1000) = 110445000 \; mm^3$

$\because \; S_{nv.}(under) > S_{nv.}(above) \quad \therefore \; Z > 150 \; mm$

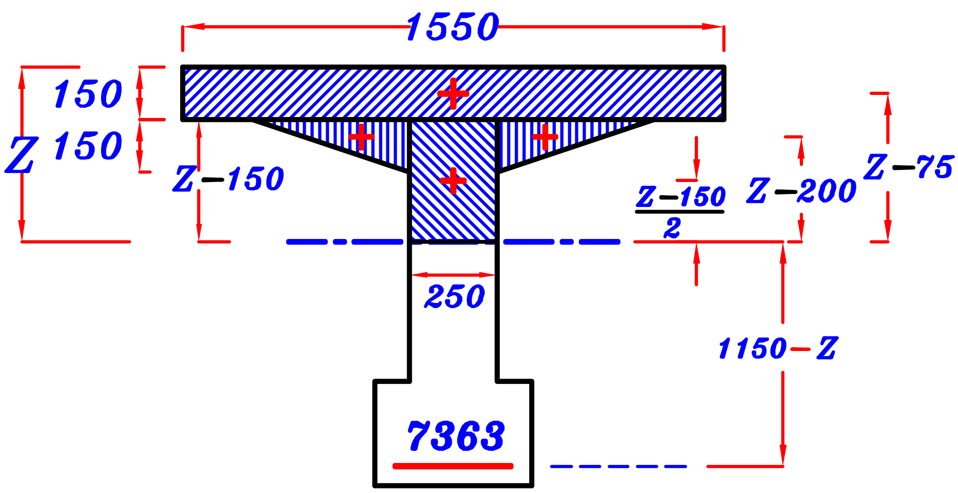


$S_{nv.}(above) = 150 * 1550 * (225) + 250 * 150 * (75)$

$+ \; 2 * (0.5 * 450 * 150) \; (100) = 61875000 \; mm^3$

$S_{nv.}(under) = 15 * 7363 * (850) = 93878250 \; mm^3$

$\because \; S_{nv.}(under) > S_{nv.}(above) \quad \therefore \; Z > 300 \; mm$



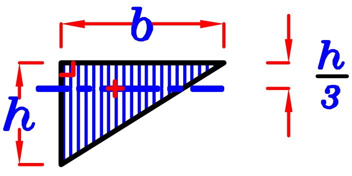
① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\,above\,(N.A.)} = S_{nv.\,under\,(N.A.)}}$

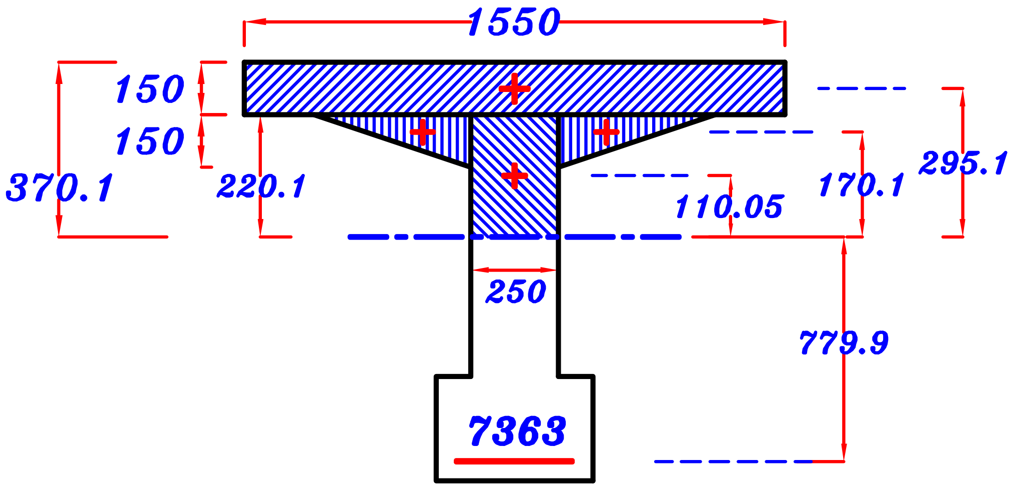
$$(1550)(150)(Z-75) + (250)(Z-150)\left(\frac{Z-150}{2}\right) + 2 * (0.5 * 450 * 150)\,(Z-200)$$

$$= (15)(7363)(1150-Z)$$

$$\boxed{Z = 370.1 \; mm}$$



$$I_x = \frac{b\,h^3}{36}$$



③ $$I_{nv} = \frac{1550(150)^3}{12} + (1550)(150)(295.1)^2 + \frac{250\,(220.1)^3}{3} + 2*\frac{450*150^3}{36}$$

$$+\,2*(0.5*450*150)\,(170.1)^2 + (15)(7363)(779.9)^2 = 90786444070 \ mm^4$$

④ $$M_{wc} = \frac{\frac{2}{3}F_c * I_{nv}}{Z} \quad \text{-------} \quad \text{as T-Sec.}$$

$$= \frac{(\frac{2}{3})\,10.5 * 90786444070}{370.1} = 1717117289 \ N.mm$$

$$= 1717.1 \ kN.m$$

⑤ $$M_{ws} = \frac{(\frac{F_s}{n}) * I_{nv}}{d - Z} = \frac{(\frac{220}{15}) * 90786444070}{1150 - 370.1} = 1707314416 \ N.mm$$

$$= 1707.3 \ kN.m$$

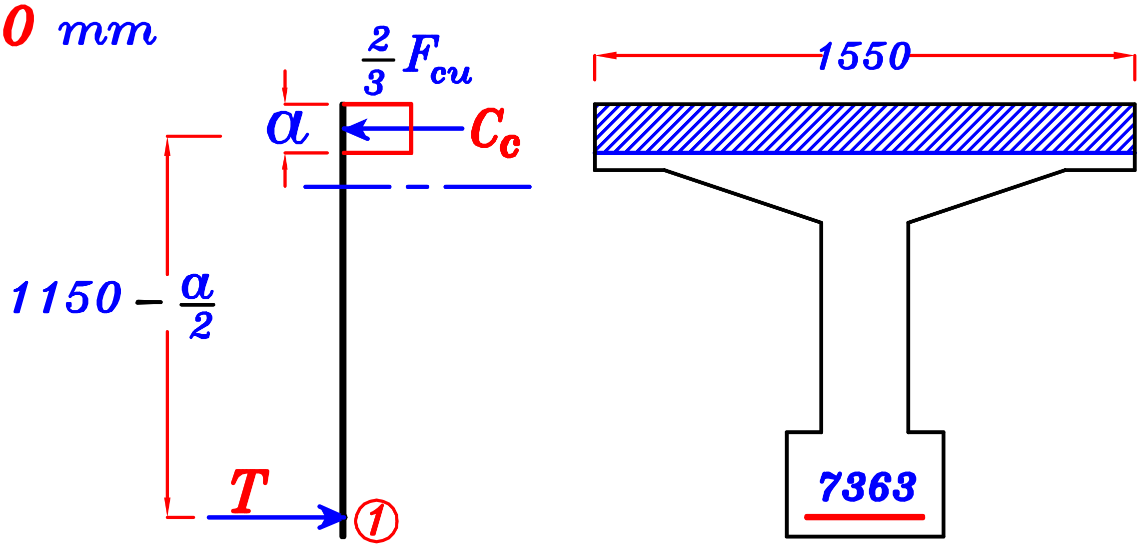
⑥ $$\boxed{M_w = 1707.3 \ kN.m}$$

c – The Failure Moment. (M_{ult})

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 400} * 1150 = 690 \text{ mm}$$

② Assume $a \leqslant t_s$

$a < 150$ mm



③ From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(1550) = (7363)(400) \longrightarrow a = 95.0 \text{ mm} < t_s \quad \therefore \text{O.K.}$$

$$\therefore C = 1.25\, a = 1.25 * 95.0 = 118.75 \text{ mm} < C_b$$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B \left(d - \frac{a}{2}\right)$$

$$= \frac{2}{3}(30)(95.0)(1550)\left(1150 - \frac{95.0}{2}\right) = 3246862500 \text{ N.mm}$$

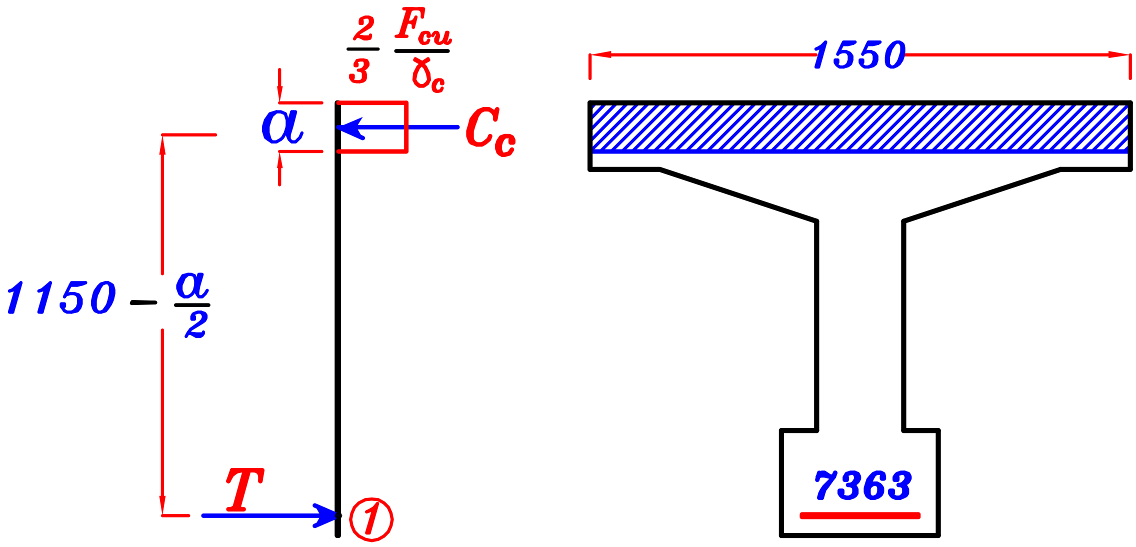
$$\therefore \boxed{M_{ult} = 3246.86 \text{ kN.m}}$$

d- The Ultimate Limit Moment. ($M_{U.L.}$)

$$a_{min} = 0.1\,d = 0.1 * 1150 = 115 \text{ mm}$$

$$a_{max} = 0.8\left(\frac{2}{3}\right)\left[\frac{600}{600 + (F_y \backslash \gamma_s)}\right] * d = 0.337\,d = 0.337 * 1150 = 387.55 \text{ mm}$$

assume $a \leqslant t_s$ $\quad a < 150$ mm



From equilibrium eqn. $\frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * F_s$ ---- a , F_s

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\therefore \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3}\left(\frac{30}{1.5}\right)(a)\,(1550) = (7363)\left(\frac{400}{1.15}\right)$$

$\longrightarrow a = 123.92$ mm $< t_s$ $\quad \therefore$ o.k.

$\therefore a_{min} < a < a_{max}$ $\quad \therefore$ right assumption $F_s = \frac{F_y}{\gamma_s}$

$$\therefore M_{U.L.} = \frac{2}{3}\frac{F_{cu}}{\gamma_c} * a * B\left(d - \frac{a}{2}\right)$$

$$= \frac{2}{3}\left(\frac{30}{1.5}\right)(123.92)\,(1550)\left(1150 - \frac{123.92}{2}\right) = 2786484947 \text{ N.mm}$$

$$= 2786.48 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 2786.48 \text{ kN.m}}$$

- The Factor Of Safty For Loads.

$$= \left(\frac{M_{U.L.}}{M_w}\right) = \frac{2786.48}{1707.3} = 1.63$$

- The Factor Of Safty For Material.

$$= \left(\frac{M_{ult}}{M_{U.L.}}\right) = \frac{3246.86}{2786.48} = 1.165$$

- The Global Factor Of Safty.

$$= \left(\frac{M_{ult}}{M_w}\right) = \frac{3246.86}{1707.3} = 1.90$$

Example.

Data. $F_{cu} = 25\ N/mm^2$

st. 360/520

Req.

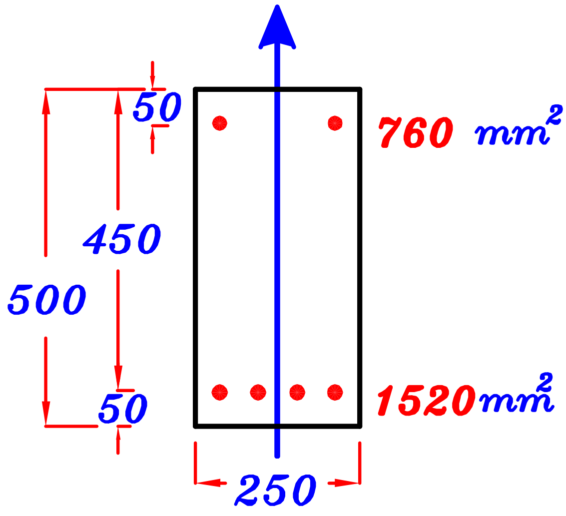
For the shown Cross-Section

1- Calculate $M_{cr.}$

2- Calculate M_w

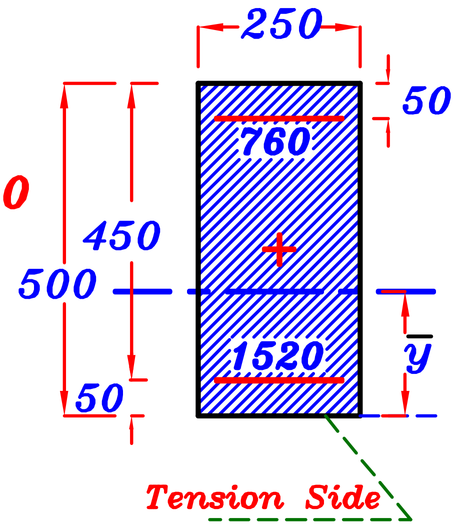
3- Calculate M_{ult}

4- Calculate $M_{U.L.}$



$$\frac{A_s'}{A_s} > 0.2 \longrightarrow \text{don't neglect } A_s'$$

Solution. 1- $M_{cr.}$



① $n = \dfrac{E_s}{E_c} = \dfrac{2*10^5}{4400\sqrt{25}} = 9.09 \longrightarrow n = 10$

② $A_v = b*t + (n-1)A_s + (n-1)A_s'$

$A_v = 250*500 + (10-1)(1520) + (10-1)(760)$

$= 145520\ mm^2$

③ $\bar{y}_t = \dfrac{250*500*250 + (10-1)(1520)(50) + (10-1)(760)(450)}{145520} = 240.6\ mm$

④ $I_{gross} = \dfrac{250*500^3}{12} + 250*500(250-240.6)^2 + (10-1)(1520)(240.6-50)^2$

$+ (10-1)(760)(450-240.6)^2 = 3412106414\ mm^4$

⑤ $F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{25} = 3.0\ N\backslash mm^2$

⑥ $M_{cr} = \dfrac{F_{ctr} * I_g}{\bar{y}_t} = \dfrac{3.0 * 3412106414}{240.6} = 42544967.7\ N.mm$

$= \dfrac{42544967.7\ N.mm}{10^6} = 42.54\ kN.m$

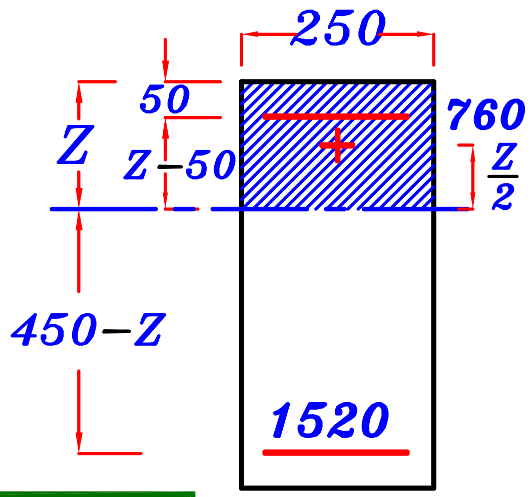
$$\boxed{M_{cr} = 42.54\ kN.m}$$

2- M_w

Allowable stresses

$F_{cu} = 25 \ N\backslash mm^2 \longrightarrow F_c = 9.5 \ N\backslash mm^2$

$F_y = 360 \ N\backslash mm^2 \longrightarrow F_s = 200 \ N\backslash mm^2$



① Take $n = 15$

② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) + (n-1)\,A_{s`}\,(Z - d`) = n\,A_s\,(d - Z)$$

$$250\,(Z)\left(\frac{Z}{2}\right) + (14)(760)(Z - 50) = (15)(1520)(450 - Z)$$

$$\boxed{Z = 189.1 \ mm}$$

③ Get $I_{nv} = \dfrac{bZ^3}{3} + (n-1)\,A_{s`}\,(Z - d`)^2 + n\,A_s\,(d - Z)^2$

$$I_{nv} = \frac{250\,(189.1)^3}{3} + (14)(760)(189.1 - 50)^2 + (15)(1520)(450 - 189.1)^2$$

$$= 2321339454 \ mm^4$$

④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{9.5 * 2321339454}{189.1} = 116619380 \ N.mm$

$= 116.62 \ kN.mm$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 2321339454}{450 - 189.1}$

$= 118632398 \ N.mm = 118.6 \ kN.m$

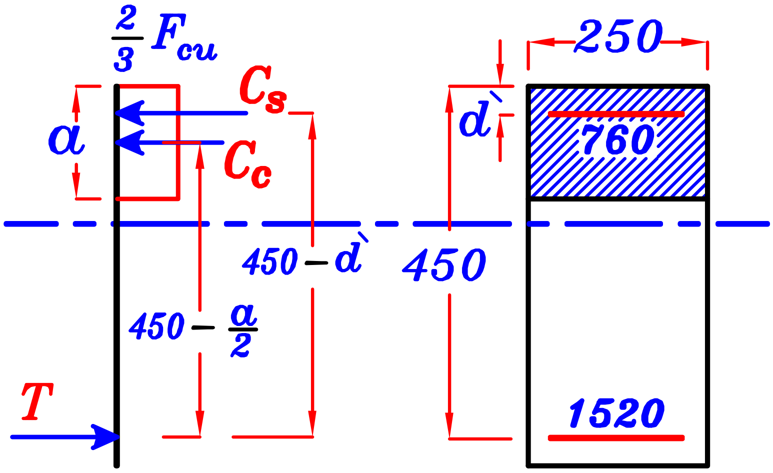
⑥ $\boxed{M_w = 116.62 \ kN.m}$

3- M_{ult}

Take للتسهيل

$F_{s`}$ (For compression steel) $= F_y$

$C_s = A_{s`} * F_y$



① $C_b = \dfrac{600}{600 + F_y} * d = \dfrac{600}{600 + 360} * 450 = 281.2$ mm

② From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} F_{cu} * a * b + A_{s`} * F_y = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(25)(a)(250) + (760)(360) = (1520)(360) \longrightarrow a = 65.6 \text{ mm}$$

$\therefore$ $C = 1.25\,a = 1.25 * 65.6 = 82.0$ mm $< C_b$

$\therefore$ **The Section is Under Reinforced Sec.**

and the assumption is right $F_s = F_y$

M_{ult} = The moment about the steel.

$$M_{ult} = C_c * (d - \frac{a}{2}) + C_s * (d - d`)$$

$$= \frac{2}{3} F_{cu} * a * b \; (d - \frac{a}{2}) + A_{s`} * F_y \; (d - d`)$$

$$= \frac{2}{3}(25)(65.6)(250)\left(450 - \frac{65.6}{2}\right) + (760)(360)(450 - 50)$$

$= 223474666$ N.mm $= 223.47$ kN.m

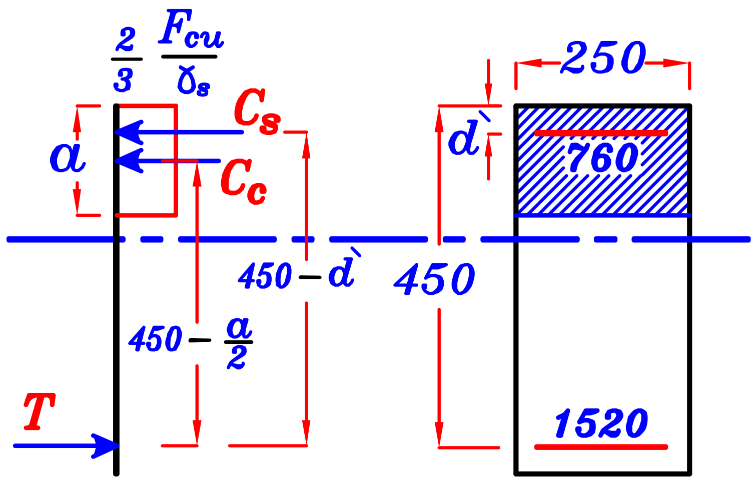
$$\boxed{M_{ult} = 223.47 \text{ kN.m}}$$

4- $M_{U.L.}$

Take للتسهيل

$F_{s}`$ (For compression steel) $= \frac{F_y}{\gamma_s}$

$C_s = A_s` * \frac{F_y}{\gamma_s}$



$a_{min} = 0.1\ d = 0.1 * 450 = 45$ mm

$a_{max} = 0.8 \left(\frac{2}{3}\right) \left[\frac{600}{600 + (F_y \backslash \gamma_s)} \right] * d = 0.35\ d = 0.35 * 450 = 157.5$ mm

From equilibrium eqn. $C_c + C_s = T$

$$\frac{2}{3} \frac{F_{cu}}{\gamma_s} * (a * b) + A_s` * \frac{F_y}{\gamma_s} = A_s * F_s \quad ---- \quad a\ ,\ F_s$$

assume $F_s = \frac{F_y}{\gamma_s}$ (Under reinforced Sec.)

$$\frac{2}{3} \frac{F_{cu}}{\gamma_s} * (a * b) + A_s` * \frac{F_y}{\gamma_s} = A_s * \frac{F_y}{\gamma_s}$$

$$\frac{2}{3} \left(\frac{25}{1.5}\right)(a)\ (250) + (760) \left(\frac{360}{1.15}\right) = (1520) \left(\frac{360}{1.15}\right)$$

$\longrightarrow a = 85.64$ mm $\qquad \therefore a_{min} < a < a_{max}$

$\therefore$ right assumption

$M_{U.L.}$ = The moment about the steel.

$$M_{U.L.} = C_c * (d - \frac{a}{2}) + C_s * (d - d`)$$

$$= \frac{2}{3} \frac{F_{cu}}{\gamma_s} * a * b\ (d - \frac{a}{2}) + A_s` * \frac{F_y}{\gamma_s} (d - d`)$$

$$= \frac{2}{3} \left(\frac{25}{1.5}\right)(85.64)(250) \left(450 - \frac{85.64}{2}\right) + (760) \left(\frac{360}{1.15}\right) (450 - 50)$$

$= 192028815$ N.mm $= 192.0$ kN.m

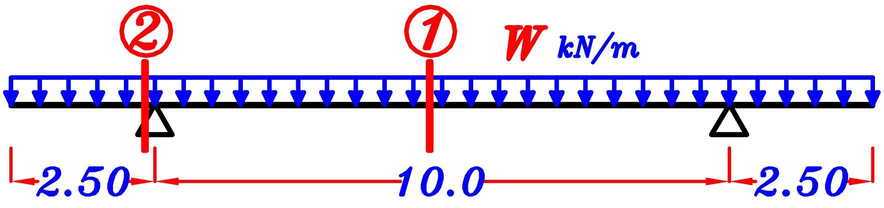
$$\boxed{M_{U.L.} = 192.0 \text{ kN.m}}$$

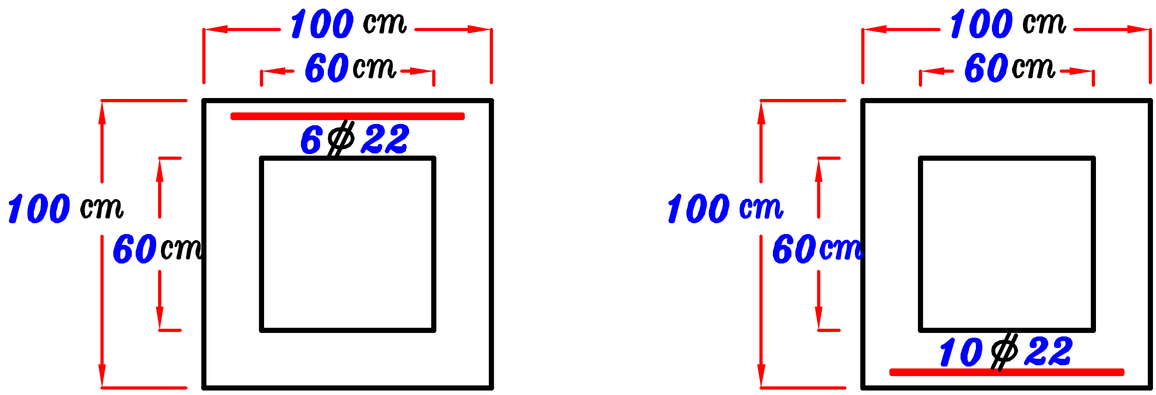
Example.

The Figure shows a statical system of an overhanging beam, subjected to uniform distributed load **(W)** with the shown sections. It is required to calculate the critical value of the load **(W)** in each of the Following cases:

1- The cracking load of the girder (Steel reinforcement can be ignored)

2- The ultimate load of the girder.

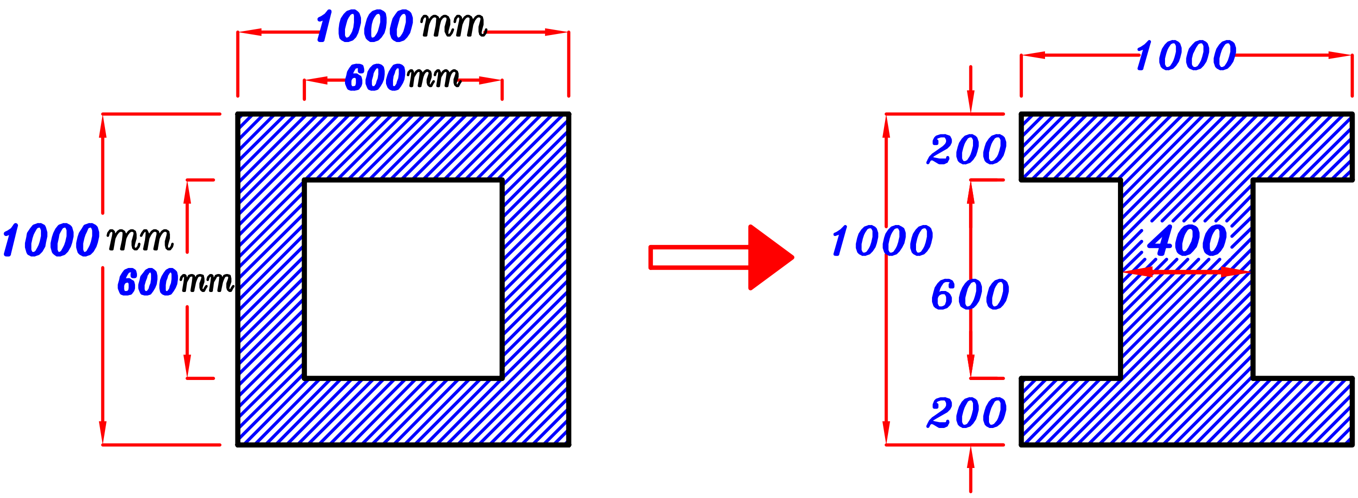




Section 2 Section 1

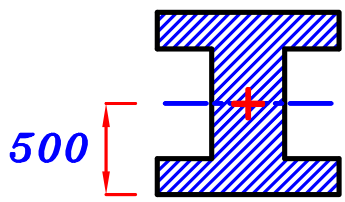
For Cracking Moment. M_{cr}

IF we neglect the steel. M_{cr} (Sec.1) = M_{cr} (Sec.2)



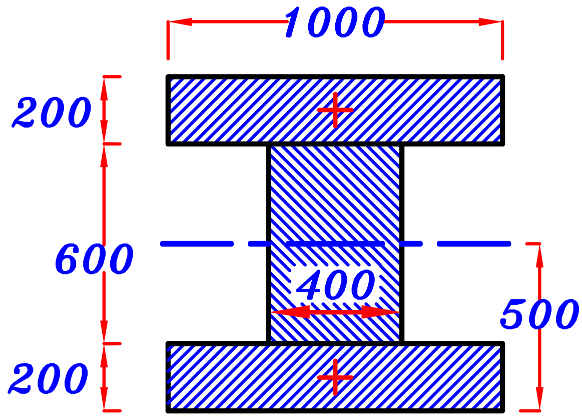
① $A_v = A_c = 2\,(200 * 1000) + 400 * 600 = 640000\ mm^2$

② $\bar{y}_t = 500\ mm$



③ $$I_g = \frac{400 * 600^3}{12} + 2\left[\frac{1000 * 200^3}{12} + 1000 * 200(500 - 100)^2\right]$$

$$= 72533333333 \ mm^4$$



④ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28 \ N\backslash mm^2$$

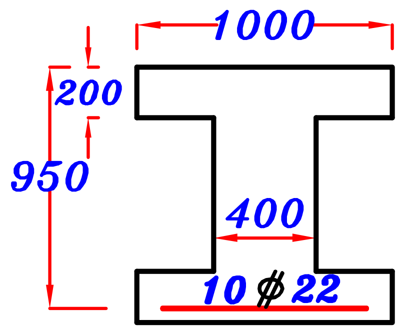
⑤ $$M_{cr} = \frac{F_{ctr} * I_g}{\overline{y}_t} = \frac{3.28 * 72533333333}{500} = 475818666 \ N.mm$$

$$= 475.8 \ kN.m$$

$$\boxed{M_{cr1} = M_{cr2} = 475.8 \ kN.m}$$

For Ultimate Moment. M_{ult}

Section 1

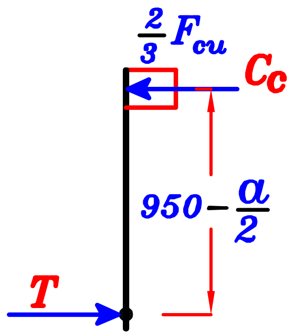


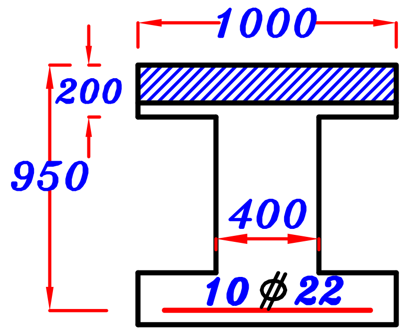
$$A_s = 10 \phi 22 = 10\left[\frac{\pi * 22^2}{4}\right] = 3801 \ mm^2$$

① $$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 950 = 593.7 \ mm$$

② Assume $a \leqslant t_s$

$a < 200 \ mm$





③ From equilibrium eqn.

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(1000) = (3801)(360) \longrightarrow a = 68.4 \ mm < t_s \quad \therefore \ O.K.$$

$$\therefore \ C = 1.25 \ a = 1.25 * 68.4 = 85.52 \ mm < C_b$$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$

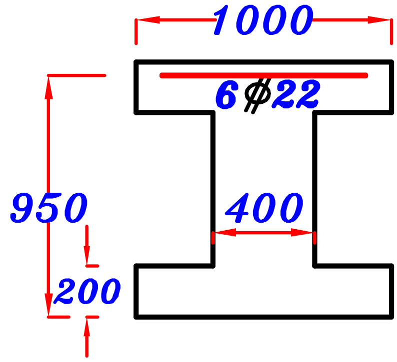
$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B\, (d - \frac{a}{2})$$

$$M_{ult} = \frac{2}{3}(30)(68.4)(1000)\left(950 - \frac{68.4}{2}\right) = 1252814400 \text{ N.mm} = 1252.8 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 1252.8 \text{ kN.m}}$$

Section 2

$$A_s = 6\,\phi 22 = 6\left[\frac{\pi * 22^2}{4}\right] = 2280 \text{ mm}^2$$

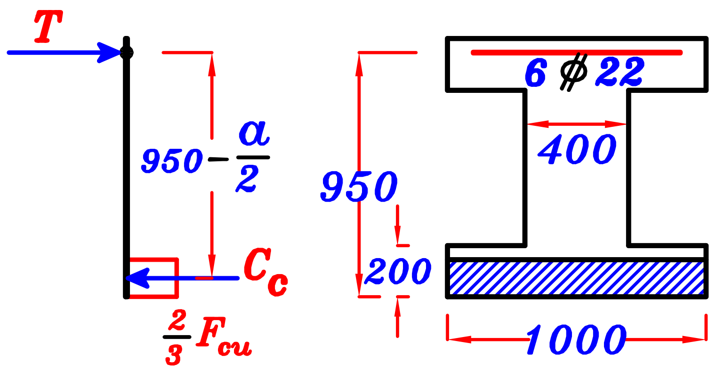


① $$C_b = \frac{600}{600 + F_y} * d$$

$$= \frac{600}{600 + 360} * 950 = 593.7 \text{ mm}$$

② Assume $a \leqslant t_s$

$a < 200$ mm



③ From equilibrium eqn.

$$\frac{2}{3} F_{cu} * a * B = A_s * F_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(1000) = (2280)(360) \longrightarrow a = 41.04 \text{ mm} < t_s \quad \therefore \text{O.K.}$$

$$\therefore C = 1.25\, a = 1.25 * 41.04 = 51.3 \text{ mm} < C_b$$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

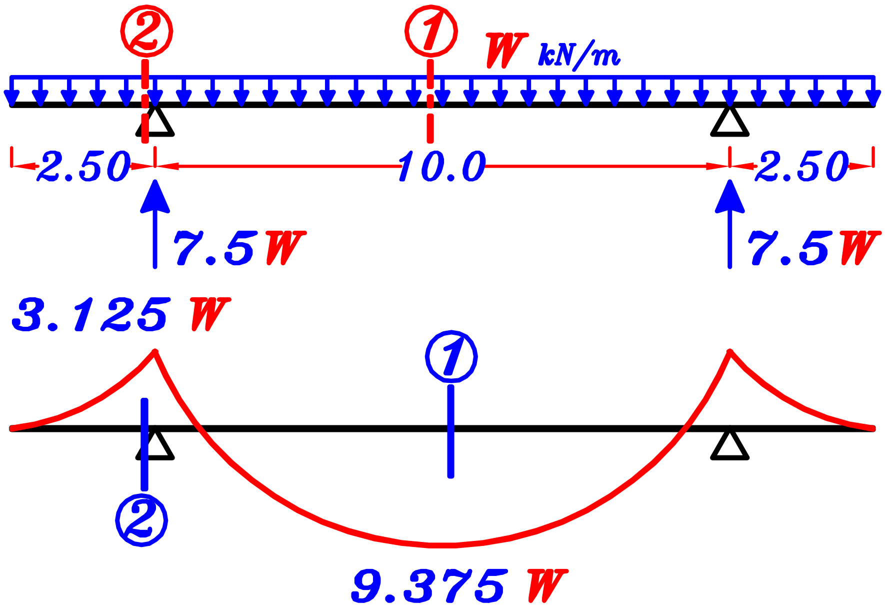
and the assumption is right $F_s = F_y$

$$\therefore M_{ult} = \frac{2}{3} F_{cu}\, a\, B\, (d - \frac{a}{2})$$

$$\therefore M_{ult} = \frac{2}{3}(30)(41.04)(1000)\left(950 - \frac{41.04}{2}\right) = 762917184 \text{ N.mm} = 762.9 \text{ kN.m}$$

$$\therefore \boxed{M_{ult} = 762.9 \text{ kN.m}}$$

Actual Moment.



1– The cracking load of the girder. (W_{cr})

Sec. ① $M_{act.} = 9.375\ W$ $\qquad M_{cr1} = 475.8\ kN.m$

$\therefore\ 9.375\ W_{cr.} = 475.8\ kN.m \longrightarrow W_{cr.1} = 50.75\ kN/m$

Sec. ② $M_{act.} = 3.125\ W$ $\qquad M_{cr2} = 475.8\ kN.m$

$\therefore\ 3.125\ W_{cr.} = 475.8\ kN.m \longrightarrow W_{cr.2} = 152.2\ kN/m$

$$\boxed{W_{cr.} = 50.75\ kN/m}$$

1– The ultimate load of the girder. (W_{ult})

Sec. ① $M_{act.} = 9.375\ W$ $\qquad M_{ult1} = 1252.8\ kN.m$

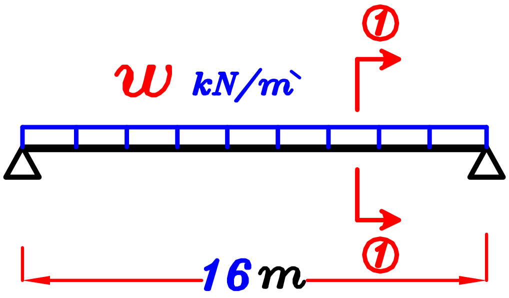
$\therefore\ 9.375\ W_{ult} = 1252.8\ kN.m \longrightarrow W_{ult1} = 133.6\ kN/m$

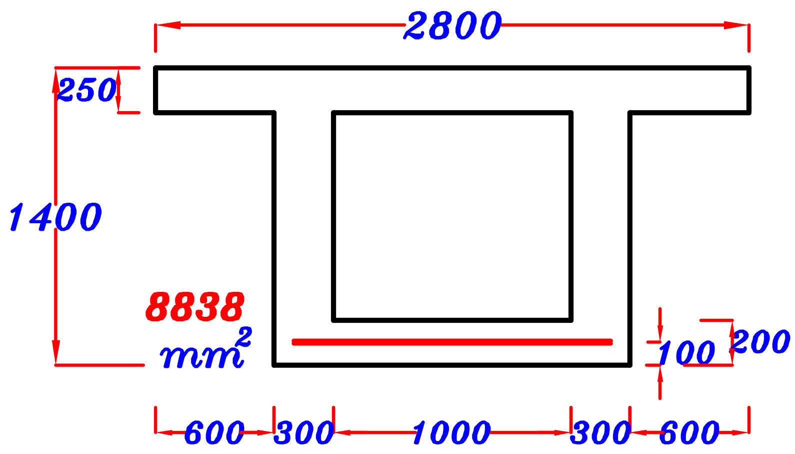
Sec. ② $M_{act.} = 3.125\ W$ $\qquad M_{ult2} = 762.9\ kN.m$

$\therefore\ 3.125\ W_{ult} = 762.9\ kN.m \longrightarrow W_{ult2} = 244.12\ kN/m$

$$\boxed{W_{ult} = 133.6\ kN/m}$$

Example.





Sec. (1−1)

Data.

$F_{cu} = 30 \quad N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2$

$F.C. = 3.50 \quad kN\backslash m^2$

$$\boxed{A_s = 8838 \ mm^2}$$

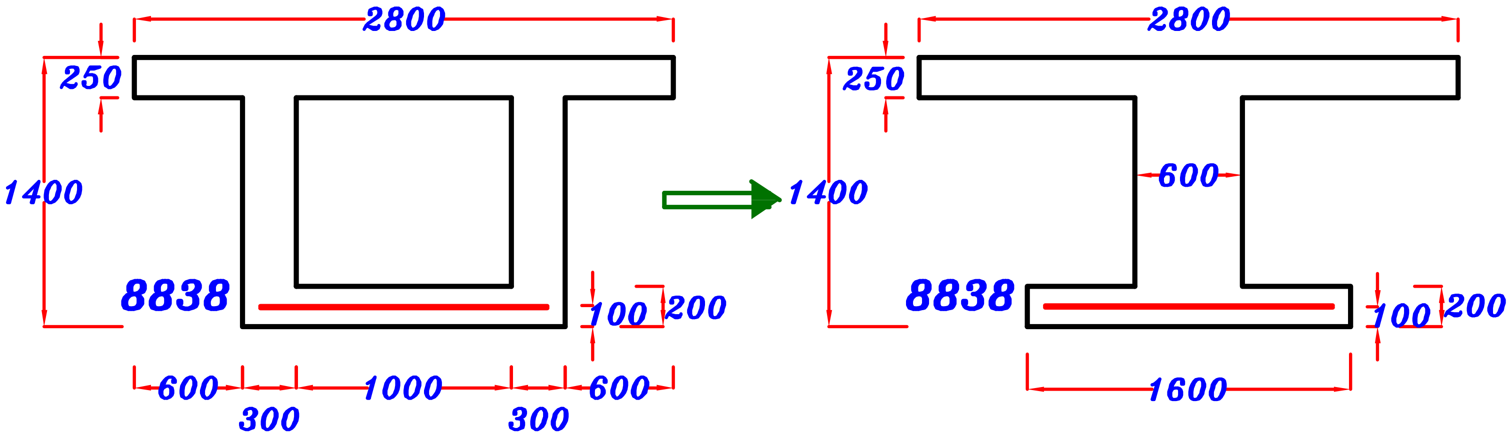
Req.

Fined the allowable working live load $(kN\backslash m^2)$

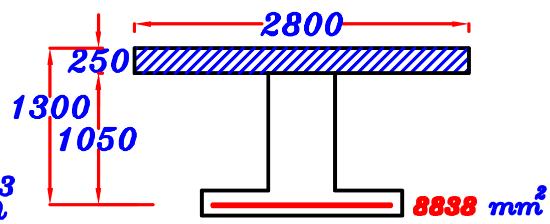
Allowable stresses

$F_{cu} = 30 \quad N\backslash mm^2 \longrightarrow F_c = 10.5 \ N\backslash mm^2$

$F_y = 360 \quad N\backslash mm^2 \longrightarrow F_s = 200 \quad N\backslash mm^2$



To know if Z is bigger or smaller than the Flange thickness $= 250$ mm

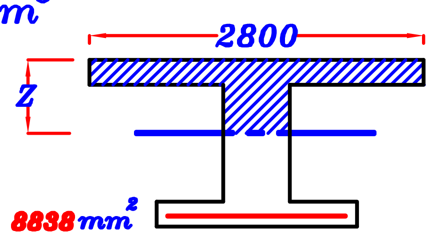


$S_{nv.}(above) = 250 * 2800 * (125) = 87500000 \ mm^3$

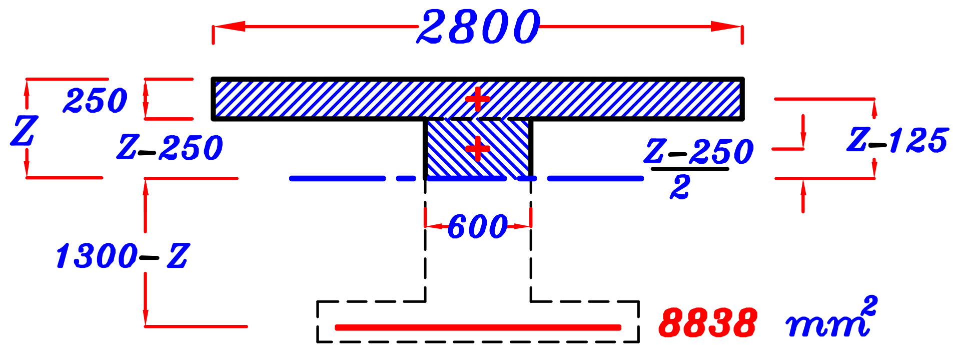
$S_{nv.}(under) = 15 * 8838 * (1050) = 139198.5 \ mm^3$

$\because \ S_{nv.}(under) > S_{nv.}(above)$

$\therefore \ Z > 250 \quad mm$



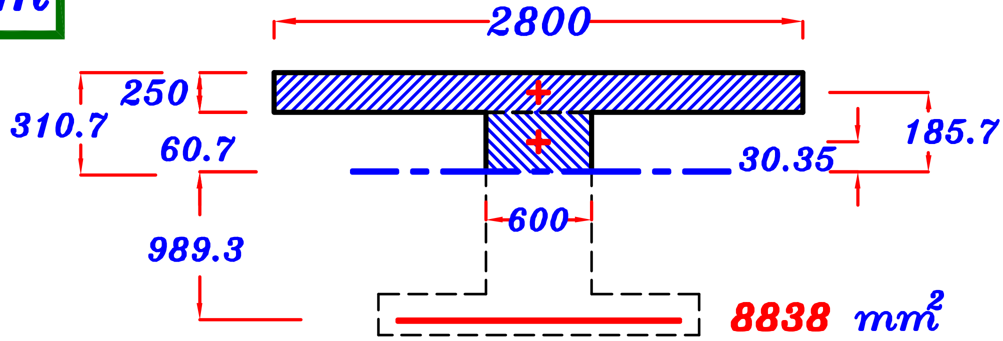
① Take $n = 15$



② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$(2800)(250)(Z-125) + (600)(Z-250)\left(\frac{Z-250}{2}\right) = (15)(8838)(1300-Z)$$

$$\boxed{Z = 310.7 \ mm}$$



③ $$I_{nv} = \frac{2800(250)^3}{12} + (2800)(250)(185.7)^2 + \frac{600(60.7)^3}{3} + (15)(8838)(989.3)^2 = 157577886000 \ mm^4$$

④ $$M_{wc} = \frac{F_c * I_{nv}}{Z}$$ ------- not as T_Sec.

$$= \frac{10.5 * 157577886000}{310.7} = 5325290643 \ N.mm$$

$$= 5325.3 \ kN.m$$

⑤ $$M_{ws} = \frac{\left(\frac{F_s}{n}\right) * I_{nv}}{d-Z}$$

$$= \frac{\left(\frac{200}{15}\right) * 157577886000}{1300 - 310.7} = 2123762741 \ N.mm$$

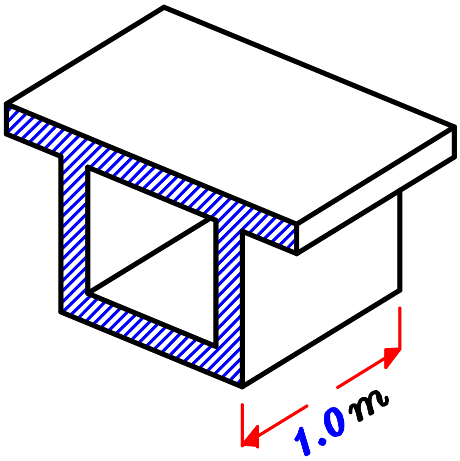
$$= 2123.7 \ kN.m$$

⑥ $$\boxed{M_w = 2123.7 \ kN.m}$$

للتحويل من $kN\backslash m^2$ إلى $kN\backslash m'$ نضرب فى العرض بالمتر

للتحويل من $kN\backslash m'$ إلى $kN\backslash m^2$ نقسم على العرض بالمتر

$w = O.W. + F.C. + L.L. = \checkmark\ kN\backslash m'$

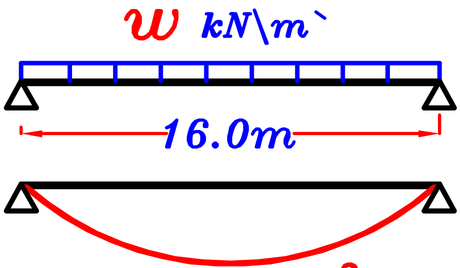


$O.W.$ of the beam For $1.0\ m.$

$= \text{Volume} * \gamma_c$

$= [0.25(2.8) + 0.95(0.6) + 0.20(1.6)]\ (25)$

$= 39.75\ kN\backslash m'$



$$M_{act.} = \frac{wL^2}{8} = \frac{w(16)^2}{8} = 32.0\ w$$

$$M_{act.} = \frac{wL^2}{8}$$

To get the allowable $L.L.$

$$M_{act.} = M_w$$

$32\ w = 2123.7\ kN.m \longrightarrow \boxed{w = 66.36\ kN\backslash m'}$

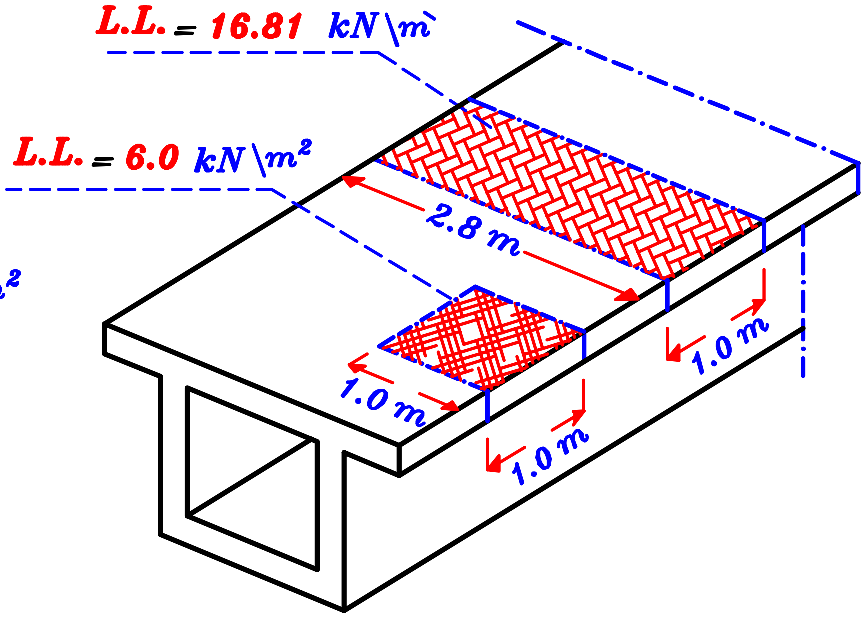
$\because\ w = O.W. + F.C. + L.L.$ العرض بالمتر

$\therefore\ 66.36 = 39.75 + (3.5 * 2.8) + L.L. \longrightarrow \boxed{L.L. = 16.81\ kN\backslash m'}$

$$L.L.\ (kN\backslash m^2) = \frac{L.L.\ (kN\backslash m')}{\text{العرض بالمتر}}$$

$$\therefore\ L.L. = \frac{16.81}{2.80} = 6.0\ kN\backslash m^2$$

$$\boxed{L.L. = 6.0\ kN\backslash m^2}$$



Example.

Figure 1 shows an elevation and cross section For a ramp path structure connecting the two levels (+7.00) and (+10.00). **It is required to:**

1- Calculate the maximum working uniform live load acting on horizontal projection which could be carried by the ramp structure (taking into consideration its own weight).

2- Calculate the Failure uniform live load of the ramp structure (taking into consideration its own weight) and state the type of Failure.

$F_{cu} = C = 30$ MPa , steel 36/52

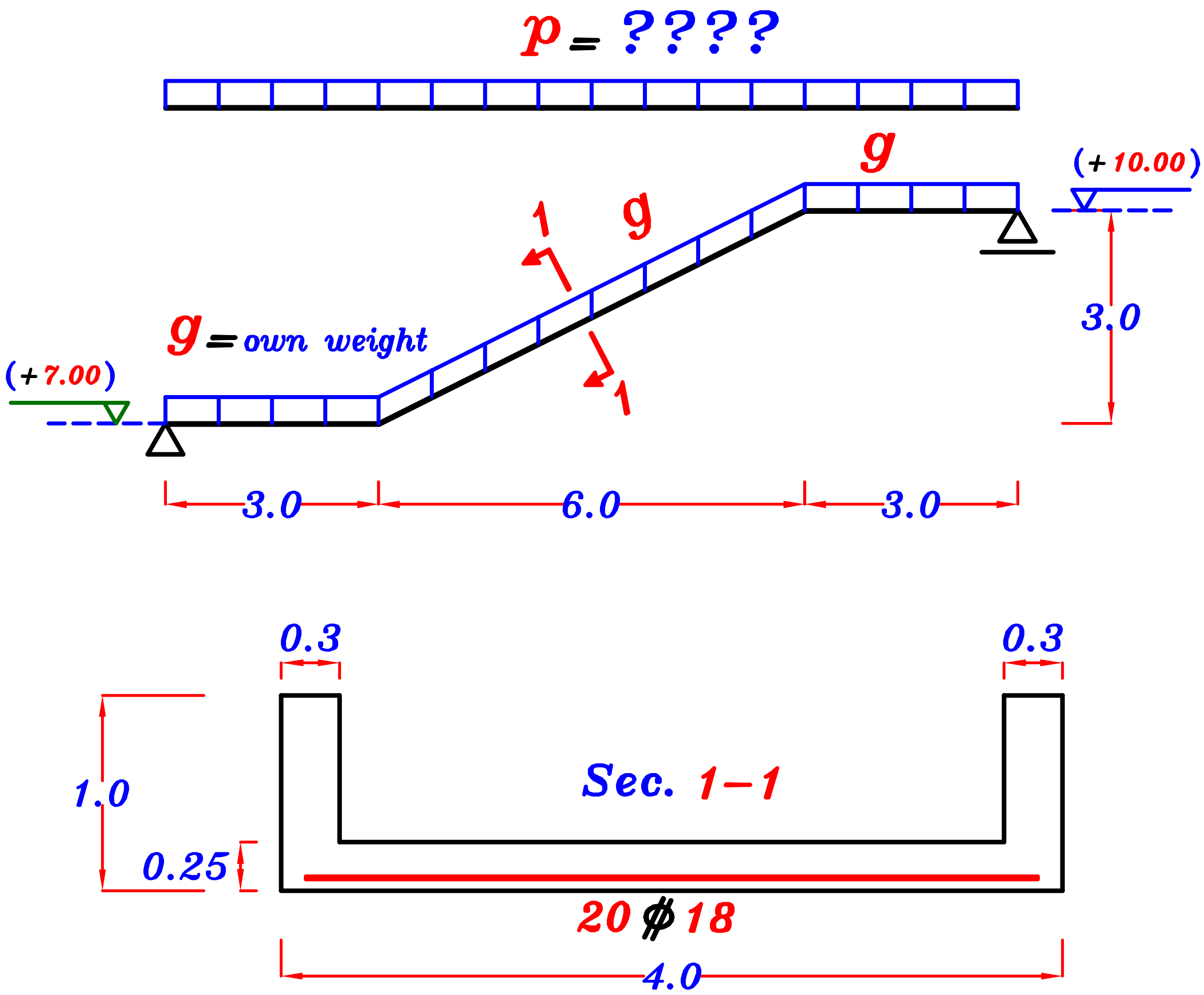
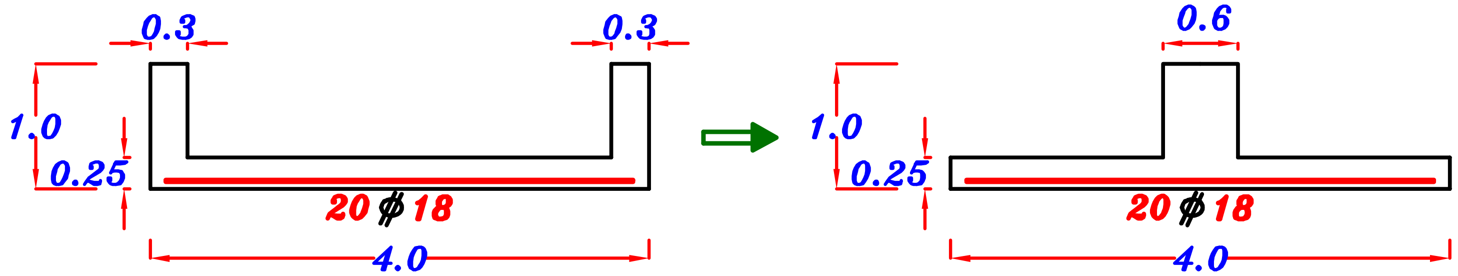
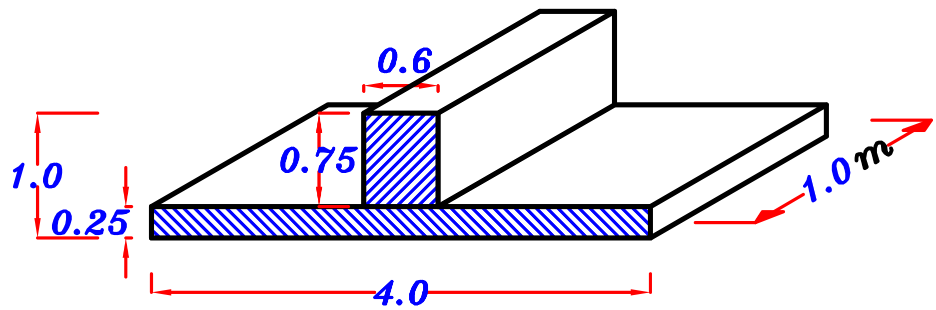


Figure 1

1 – Calculate the maximum working uniform live load acting on horizontal projection which could be carried by the ramp structure (taking into consideration its own weight).





$$o.w. = [(0.25 * 4.0) + (0.75 * 0.6)] * 1.0 * 25 = 36.25 \ kN/m$$

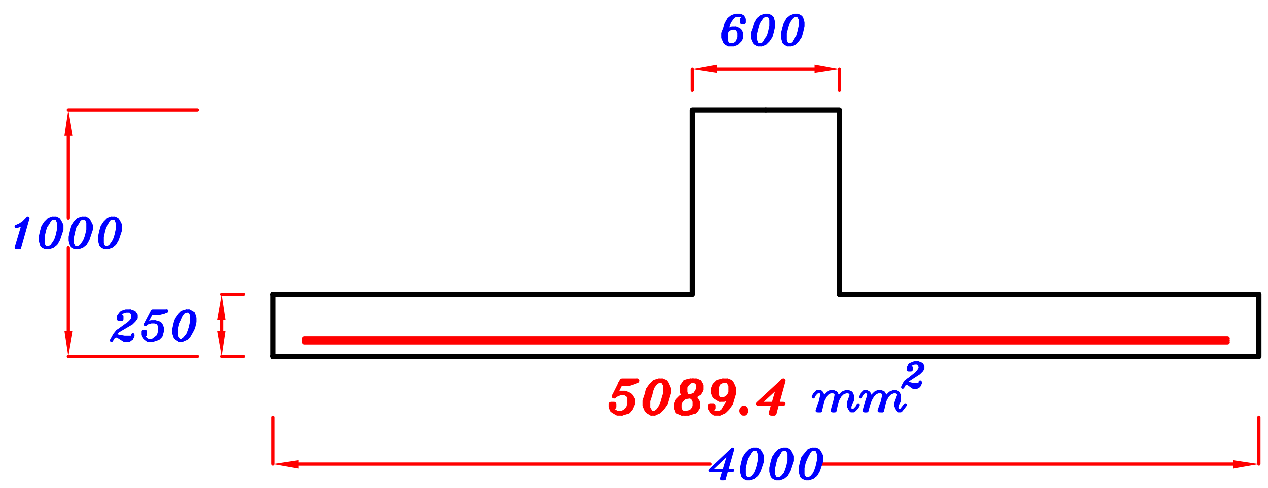
Allowable working moment. M_w

Allowable stresses

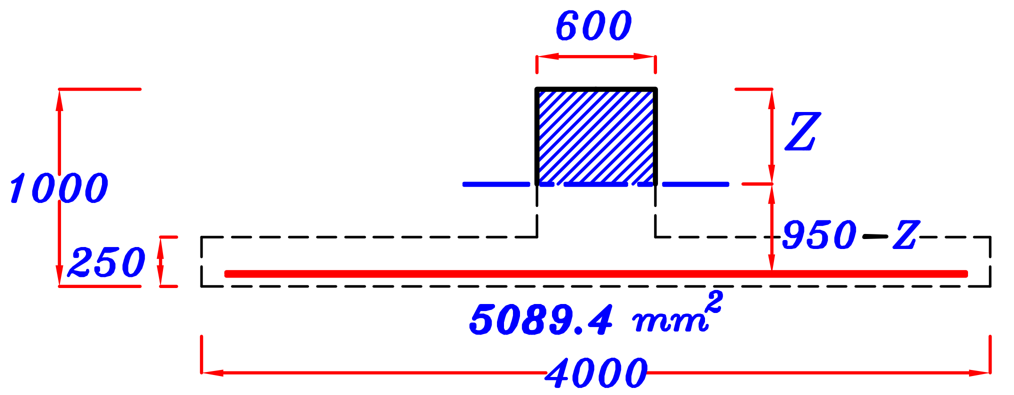
$F_{cu} = 30 \quad N \backslash mm^2 \longrightarrow F_c = 10.5 \ N \backslash mm^2$

$F_y = 360 \quad N \backslash mm^2 \longrightarrow F_s = 200 \quad N \backslash mm^2$

$$A_s = 20 \phi 18 = 20 \left[\frac{\pi * 18^2}{4}\right] = 5089.4 \ mm^2$$



① Take $n = 15$



② Get Z by taking $\boxed{S_{nv.\ above\ (N.A.)} = S_{nv.\ under\ (N.A.)}}$

$$b\,(Z)\left(\frac{Z}{2}\right) = n\,A_s\,(d - Z)$$

$$600\,(Z)\left(\frac{Z}{2}\right) = (15)\,(5089.4)\,(950 - Z)$$

$$\boxed{Z = 380.6\ mm}$$

③ Get $I_{nv} = \dfrac{b Z^3}{3} + n\,A_s\,(d - Z)^2$

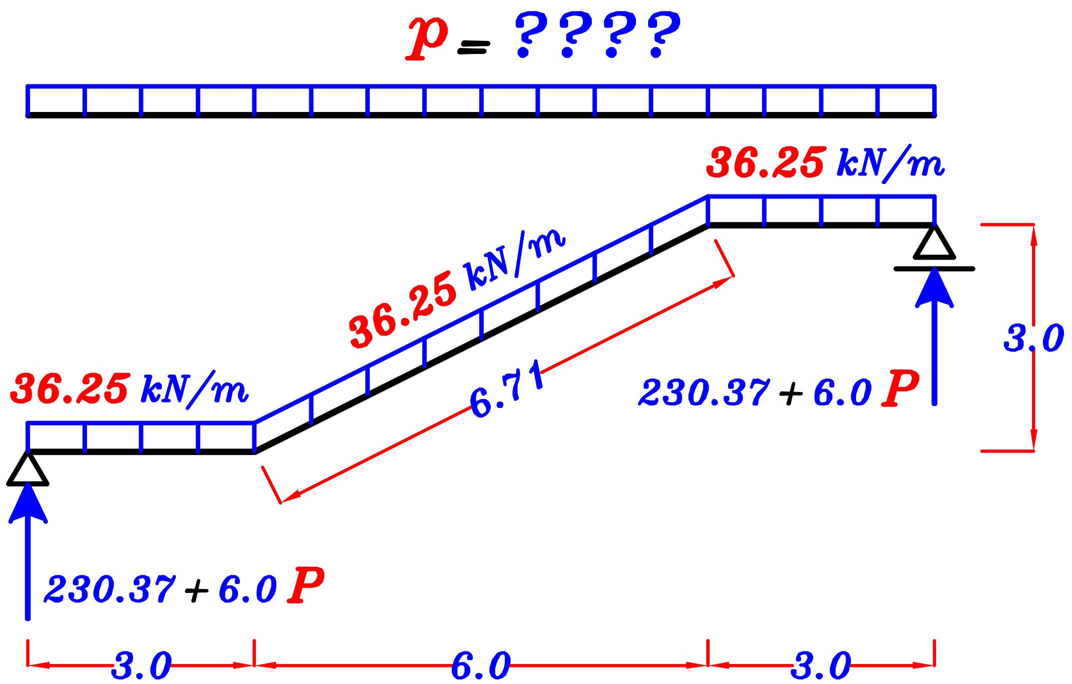
$$I_{nv} = \frac{600\ (380.6)^3}{3} + (15)\,(5089.4)\,(950 - 380.6)^2 = 35777467260\,mm^4$$

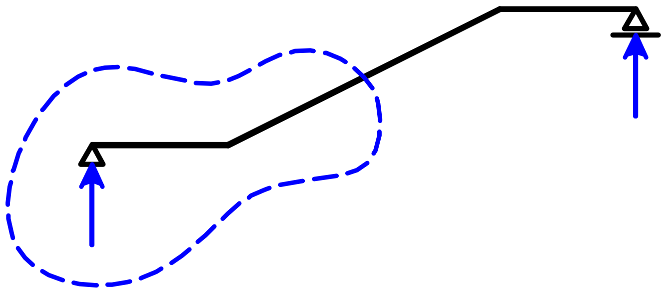
④ $M_{wc} = \dfrac{F_c * I_{nv}}{Z} = \dfrac{10.5 * 35777467260}{380.6} = 987029443\ N.mm$
$= 987.03\ kN.m$

⑤ $M_{ws} = \dfrac{\left(\frac{F_s}{n}\right) * I_{nv}}{d - Z} = \dfrac{\left(\frac{200}{15}\right) * 35777467260}{950 - 380.6} = 837781694\ N.mm$
$= 837.78\ kN.m$

⑥ $\boxed{M_{w_{all}} = 837.78\ kN.m}$

Actual working moment.





moment at mid span.

$$(230.37 + 6.0\,P)(6.0) - (36.25 * 3.0)(4.5) - (36.25 * \frac{6.71}{2})(1.5)$$

$$-(P * 6.0)(3.0) = 18.0\,P + 710.41$$

$$\boxed{M_{act} = 18.0\,P + 710.41}$$

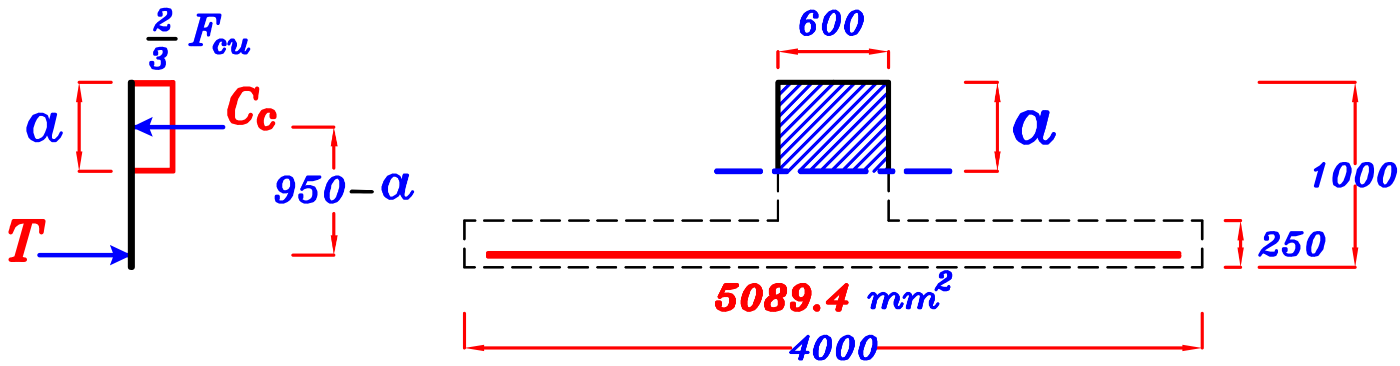
To calculate the maximum working uniform live load acting on horizontal projection.

$$M_{w_{all}} = M_{act}$$

$$837.78 = 18.0\,P_w + 710.41 \longrightarrow \boxed{P_w = 7.076\ kN/m}$$

2- Calculate the Failure uniform live load of the ramp structure (taking into consideration its own weight) and state the type of Failure.

Ultimate moment. M_{ult}



$$C_b = \frac{600}{600 + F_y} * d = \frac{600}{600 + 360} * 950 = 593.75 \text{ mm}$$

② From equilibrium eqn. $C_c = T$

$$\frac{2}{3} F_{cu} * a * b = F_s * A_s$$

Assume $F_s = F_y \longrightarrow$ (under reinforced or Balanced Sec.)

$$\frac{2}{3}(30)(a)(600) = (360)(5089.4) \longrightarrow a = 152.68 \text{ mm}$$

③ $\therefore C = 1.25\, a = 1.25 * 152.68 = 190.85 \text{ mm} < C_b$

$\therefore$ $\boxed{\text{The Section is Under Reinforced Sec.}}$

and the assumption is right $F_s = F_y$

④ By taking the moment about the steel.

$$\therefore M_{ult} = \frac{2}{3}(30)(152.68)(600)\left(950 - \frac{152.68}{2}\right)$$

$$= 1600684906 \text{ N.mm} = 1600.7 \text{ kN.m}$$

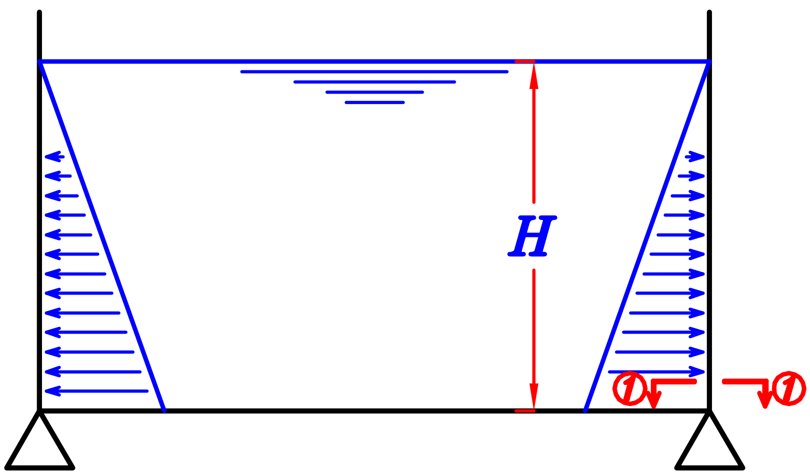
$$\boxed{M_{ult} = 1600.7 \text{ kN.m}}$$

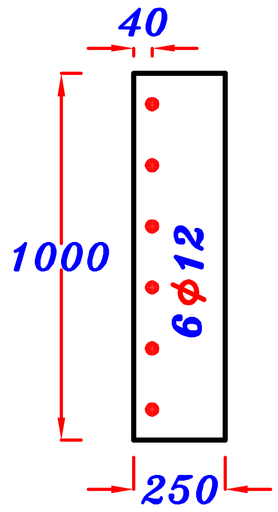
To calculate the Failure uniform live load.

$$M_{ult} = M_{act}$$

$$1600.7 = 18.0\, P_{ult} + 710.41 \longrightarrow \boxed{P_{ult} = 49.46 \text{ kN/m}}$$

Example.





Sec. (1−1)

For the given statical system & cross section of a water tank with **0.25** *m thick cantilever walls, It is required to Find the max safe height of water* **(H)** *in the tank.*

$F_{cu} = 30\ N\backslash mm^2$ st. 240/350

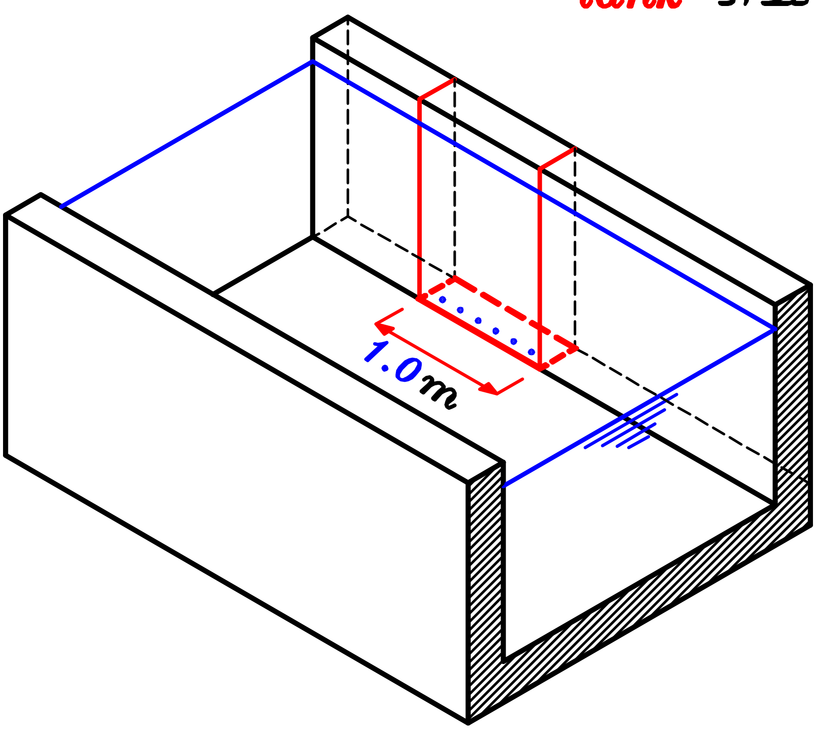
فى المنشأت المائيه يجب منع حدوث أى شروخ فى الخرسانه حتى لا تصل المياه الى حديد التسليح فيصدأ

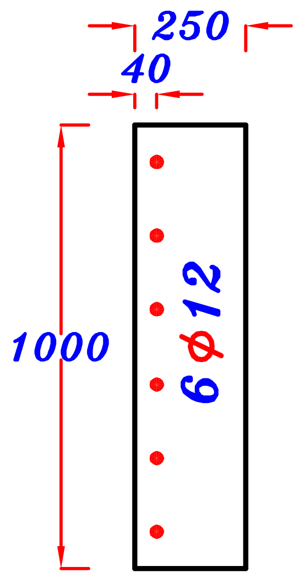
لذا أى قطاع موجود فى ال tank يجب أن لا يتعدى العزم عليه عن $M_{cr.}$

∴ لحساب أكبر ارتفاع للماء ممكن أن تتحمله حوائط ال tank

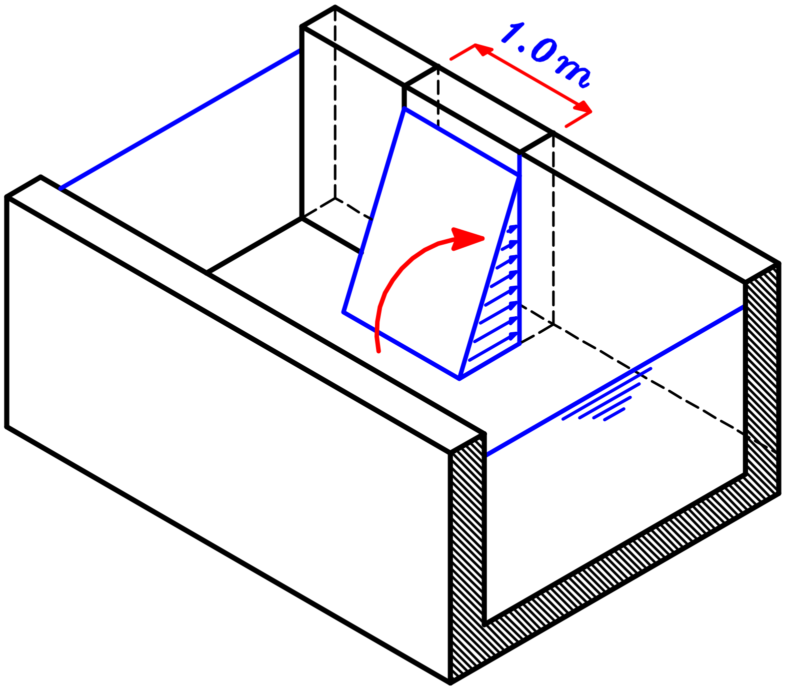
هو الارتفاع الذى يجعل العزم على القطاع السفلى للحائط مساوى تماماً لـ $M_{cr.}$

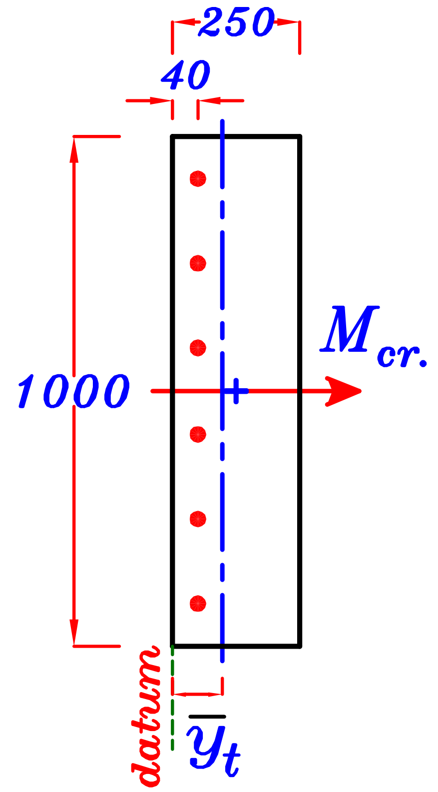
سيتم دراسه ١,٠ م من حائط ال tank





$M_{cr.}$ For the section.





$$A_s = 6\,\phi\,12 = 678.5\ mm^2$$

① $$n = \frac{E_s}{E_c} = \frac{2*10^5}{4400\sqrt{30}} = 8.29 \longrightarrow n = 10$$

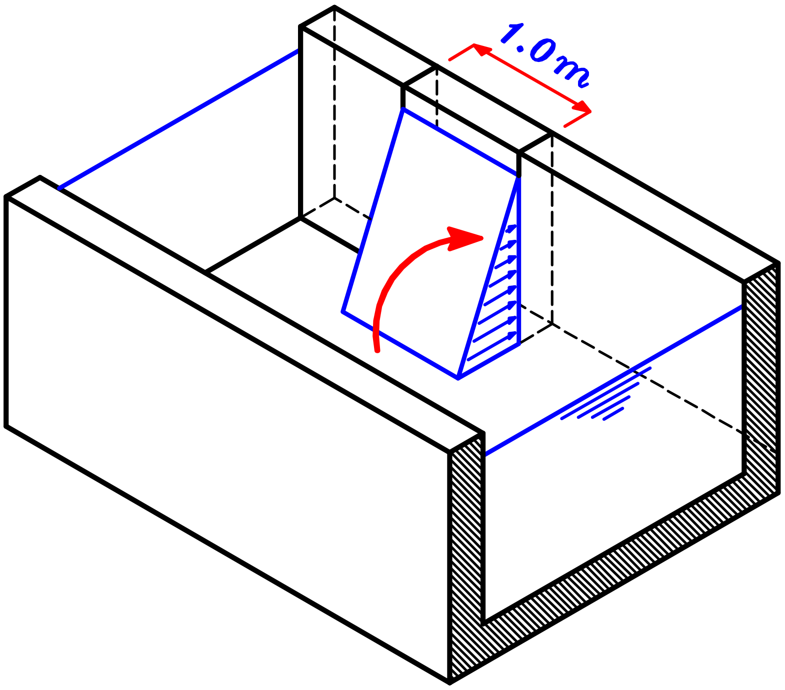
② $$A_v = 250*1000 + (10-1)(678.5) = 256106\ mm^2$$

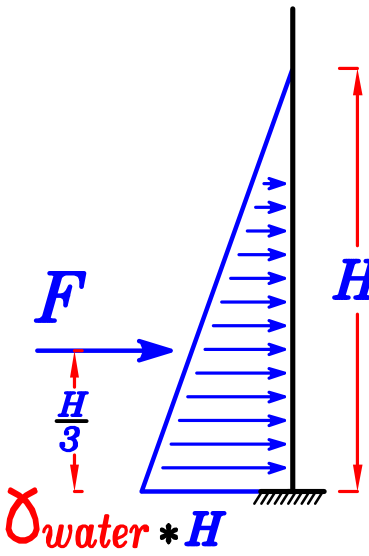
③ $$\bar{y}_t = \frac{250*1000*125 + (10-1)(678.5)(40)}{256106} = 123\ mm$$

④ $$I_g = \frac{1000*250^3}{12} + 1000*250(125-123)^2 + (10-1)(678.5)(123-40)^2$$
$$= 1345151012\ mm^4$$

⑤ $$F_{ctr} = 0.6\sqrt{F_{cu}} = 0.6\sqrt{30} = 3.28\ N\backslash mm^2$$

⑥ $$M_{cr} = \frac{F_{ctr} * I_g}{\bar{y}_t} = \frac{3.28*1345151012}{123} = 35870693.6\ N.mm$$
$$= 35.87\ kN.m$$





$$\gamma_{water} = 1.0\ t\backslash m^3 = 10.0\ kN\backslash m^3$$

$$\text{water pressure (at base)} = \gamma_{water} * H = 10\,H\ kN\backslash m^2$$

$$\text{water Force}\quad F = \tfrac{1}{2}(\gamma_{water} * H) * H = \tfrac{1}{2}(10\,H) * H = 5.0\,H^2\ kN$$

$$\text{Actual moment at Base} = F * \frac{H}{3} = 5.0\,H^2 * \frac{H}{3} = \frac{5}{3}H^3\ kN.m$$

$$\boxed{\text{Actual moment at Base} = \frac{5}{3}H^3\ kN.m}$$

To get the max. safe height (H)

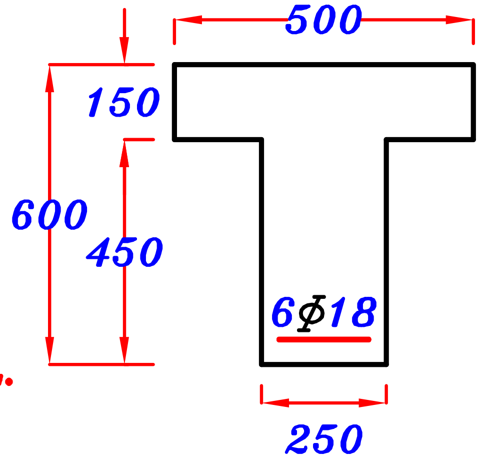
$$\therefore \text{Actual moment at Base} = M_{cr.}$$

$$\therefore \frac{5}{3}H^3 = 35.87\ kN.m \qquad \therefore \boxed{H = 2.781\ m}$$

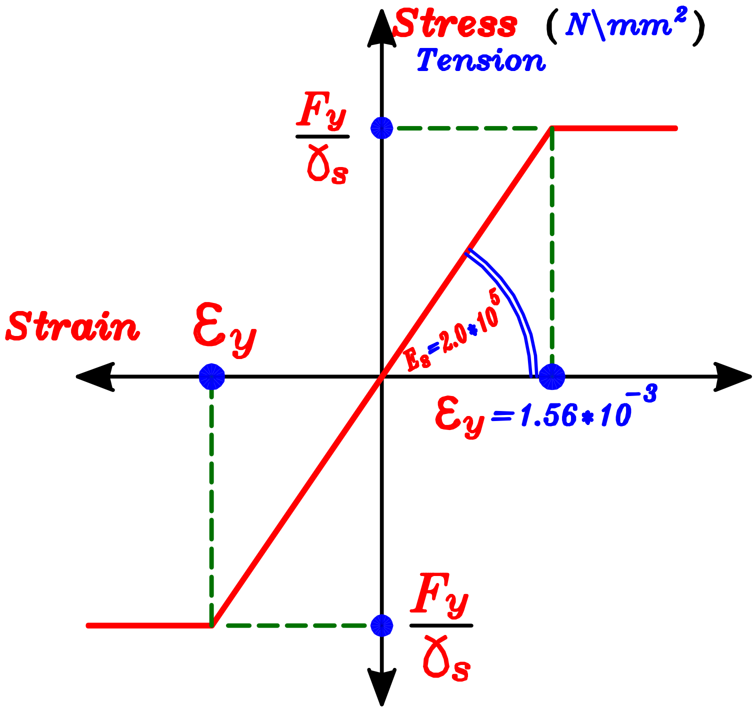
∴ اذا زاد إرتفاع الماء عن $2.781\ m$ سوف يكون العزم المؤثر على القطاع السفلى للحائط أكبر من ال $M_{cr.}$ فتتشرخ الخرسانه فيصل الماء إلى الحديد فيصدأ الحديد .

Example.

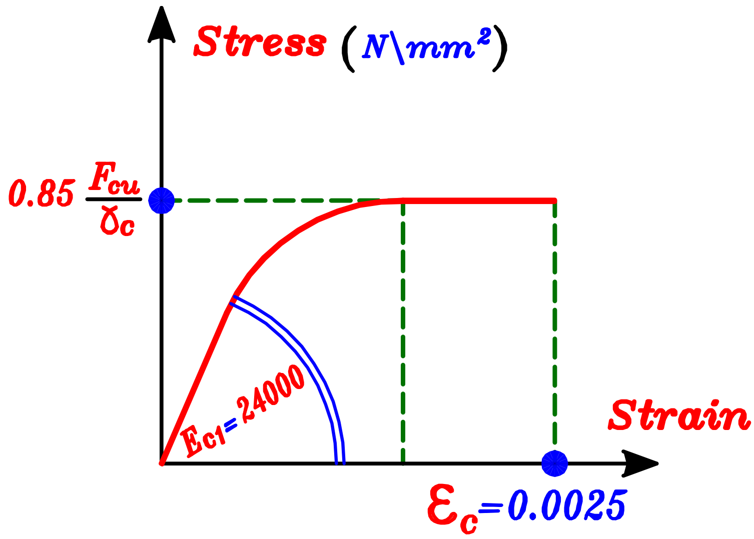
Use the data in the given Idealized Stress–Strain Curves For concrete and steel. to calculate C_b , C_{max} , a_{max} , $M_{U.L.}$ For the given section.



$F_{cu} = 25 \ N\backslash mm^2$



Idealized Stress–Strain Curve For Steel.



Idealized Stress–Strain Curve For Concrete.

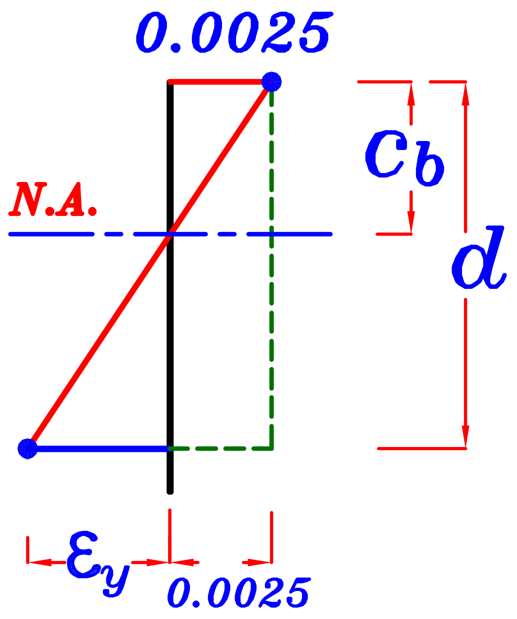
Solution.

From Curves $\varepsilon_c = 0.0025 \xrightarrow{\text{بدلاً من}} \varepsilon_c = 0.003$

max concrete stress $= 0.85 \dfrac{F_{cu}}{\delta_c} \xrightarrow{\text{بدلاً من}} \dfrac{2}{3} \dfrac{F_{cu}}{\delta_c}$

max steel stress $= \dfrac{F_y}{\delta_s} = \varepsilon_y * E_s = 1.56 * 10^{-3} * 2.0 * 10^{5} = 312 \ N\backslash mm^2$

من تشابه المثلثات



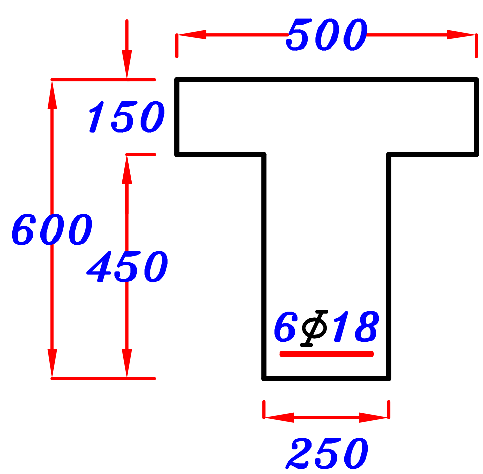
$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + \varepsilon_y}$$

$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + 1.56 * 10^{-3}} = 0.615$$

$$\therefore \boxed{c_b = 0.615\ d}$$

$$\therefore \boxed{c_{max} = \frac{2}{3}\, c_b = 0.41\, d}$$

$$\boxed{a_{max} = 0.8\, c_{max} = 0.328\, d}$$



$$A_s = 6\phi 18 = 1526\ mm^2$$

$$d = 550\ mm$$

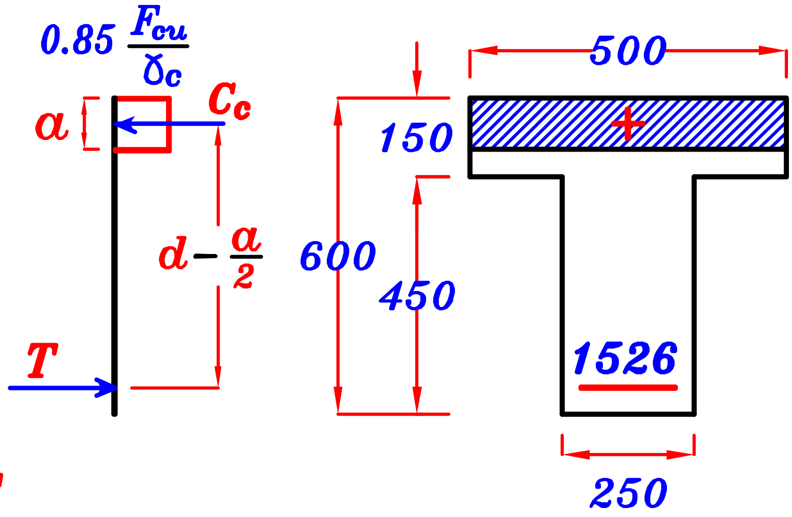
$$a_{min} = 0.1\ d = 55.0\ mm$$

$$a_{max} = 0.328\ d = 0.328 * 550$$

$$= 180.4\ mm$$

assume $a \leqslant t_s$

$a < 150$ mm



From equilibrium eqn. $C_c = T$

$$0.85 \frac{F_{cu}}{\gamma_c} * a * B = F_s * A_s \text{ ----- } a \text{ , } F_s$$

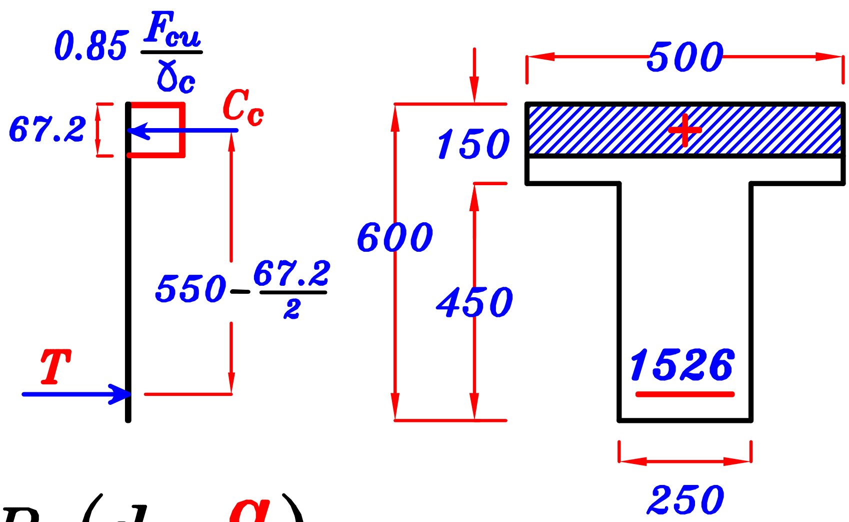
assume $F_s = \frac{F_y}{\gamma_s} = 312$ $N\backslash mm^2$ (Under reinforced Sec.)

$$\therefore \; 0.85 \left(\frac{25}{1.5}\right)(a)(500) = (312)(1526) \longrightarrow a = 67.2 \text{ mm}$$

$$\therefore \; a = 67.2 \text{ mm} < t_s \quad \therefore \text{o.k.}$$

$$a_{min} < a < a_{max} \quad \therefore \text{o.k.}$$

$$M_{U.L.} = C_c * \left(d - \frac{a}{2}\right)$$



$$M_{U.L.} = 0.85 \frac{F_{cu}}{\gamma_c} * a * B \left(d - \frac{a}{2}\right)$$

$$M_{U.L.} = 0.85 \left(\frac{25}{1.5}\right)(67.2)(500)\left(550 - \frac{67.2}{2}\right) = 245806400 \text{ N.mm}$$

$$= 245.8 \text{ kN.m}$$

$$\boxed{M_{U.L.} = 245.8 \text{ kN.m}}$$